

ABOUT THE EQUATION OF MOTION OF AN INDUSTRIAL ROBOT

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Annotation; In this article, the equation of motion of an industrial robot is formulated by kinematics.

Keywords: position function, exact problem of kinematics, cylindrical coordinate system, spherical and polar coordinate system

The main problem of kinematics is to determine the state function. The vector and coordinate transfer methods for the spatial movement of the industrial robot are effective for solving this problem. In order to solve the correct problem of kinematics about the position of the industrial robot gripper, the coordinate exchange method is usually used [1].

In forming the kinematic equation of an industrial robot, it is important to specify the technological application of the robot and the coordinate system in which its movement is presented [2].

Industrial robots are divided into the following categories according to their mechanical structure:

Robots moving in the Cartesian coordinate system are widely used in assembly, machining and arc welding. The working arm of this type of robot consists of three movable joints, the axes of which coincide with the Cartesian coordinate system [3].

Robots moving in the cylindrical coordinate system are used for assembly, machining and spot welding, pressure casting and machining.

Robots moving in a spherical and polar coordinate system are used for machine tool processing, spot and arc welding, pressure casting.

The SCARA robot is used in assembly work, loading and unloading parts on machines. This robot has a working arm consisting of a pair of parallel rotating joints.

The hinge is widely used in robotic assembly, injection molding, welding, and paint spraying. This robotic working arm should consist of at least three rotor joints.

Parallel robots are used in aviation training rooms.

The movement space of the robot is three-dimensional, and the three-dimensional square matrix does not provide the necessary information for the linear movement of its links. Therefore, the last column to facilitate operations on matrices and a $(0,0,0,1)$ vector is added to the last line, and the resulting matrix is called unidimensional matrix.

The resulting (4×4) – dimensional matrix is divided into the following four part matrices [4]:

$$A = \begin{bmatrix} R_{3 \times 3} & p_{3 \times 1} \\ f_{1 \times 3} & 1 \times 1 \end{bmatrix} = \begin{bmatrix} \text{бурилиш} & \text{силжиш} \\ \text{соддалаштирувчи вектор} & \text{масштаблaш} \end{bmatrix},$$

the orthogonal $R_{3 \times 3}$ (3×3) – matrix for the measured rotation;

$p_{3 \times 1}$ – three-dimensional displacement vector;

$f_{1 \times 3} = (0,0,0)$ vector.

If the joint is rotatable, then the rotation matrix

$$A = \begin{bmatrix} R_{3 \times 3} & 0 \\ 0 & 1 \times 1 \end{bmatrix}.$$

If intended for migration, the migration matrix

$$A = \begin{bmatrix} E_{3 \times 3} & p_{3 \times 1} \\ 0 & 1 \times 1 \end{bmatrix},$$

$$E_{3 \times 3} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

$O_i x_i y_i z_i$ coordinates from the system $O_{i-1} x_{i-1} y_{i-1} z_{i-1}$ coordinates to the system he is silent matrices at home writing possible :

$$A_i = A_i^\theta \cdot A_i^a \cdot A_i^d \cdot A_i^\alpha,$$

here

$$A_i^\theta = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos \theta_i & -\sin \theta_i & 0 \\ 0 & \sin \theta_i & \cos \theta_i & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} - x_i \text{ corner matrix } \theta_i \text{ around the axis;}$$

$$A_i^a = \begin{bmatrix} 1 & 0 & 0 & a_i \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} - x_i \text{ unit shift matrix } a_i \text{ along the axis;}$$

$$A_i^d = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} - z_{i-1} \text{ unit shift matrix } d_i \text{ along the axis;}$$

$$A_i^\alpha = \begin{bmatrix} \cos \alpha_i & -\sin \alpha_i & 0 & 0 \\ \sin \alpha_i & \cos \alpha_i & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} - z_{i-1} \text{ corner matrix } \alpha_i \text{ around the axis.}$$

industrial robot from the matrix above i - view of the matrix representing the spatial state of the link

$$A_i = \begin{bmatrix} \cos \theta_i & -\sin \theta_i \cos \alpha_i & \sin \theta_i \sin \alpha_i & a_i \cos \theta_i \\ \sin \theta_i & \cos \theta_i \cos \alpha_i & -\cos \theta_i \sin \alpha_i & a_i \sin \theta_i \\ 0 & \sin \alpha_i & \cos \alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}, (i = \overline{1, n}). \quad (1)$$

It is known that the r_i following formula is used to find the radius vector:

$$r_{i-1} = A_i r_i, \quad i = 1, 2, \dots, n.$$

Cartesian coordinate system of an arbitrary point of its working arm and the generalized coordinate system in relation to the coordinate system related to the base of the robot and represents the kinematics of the robot is as follows :

$$r_0 = A_1 A_2 \dots A_i r_i, \quad i = 1, 2, \dots, n.$$

$$\Gamma_i = A_1 A_2 \dots A_i = \begin{bmatrix} x_i & y_i & z_i & p_i \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad i = 1, 2, \dots, n.$$

$$r_0 = \Gamma_i r_i, \quad i = 1, 2, \dots, n.$$

Suppose that the A_i transition matrix represents a rotating kinematic pair, then it θ_i q_i is expressed by generalized coordinates and is variable, i.e. $\theta_i = q_i = \text{var}$.

(1) taking the time derivative of the transition matrix,

$$\frac{dA_i}{dt} = \begin{bmatrix} -\dot{q}_i \sin q_i & -\dot{q}_i \cos q_i \cos \alpha_i & \dot{q}_i \cos q_i \sin \alpha_i & -a_i \dot{q}_i \sin q_i \\ \dot{q}_i \cos q_i & -\dot{q}_i \sin q_i \cos \alpha_i & \dot{q}_i \sin q_i \sin \alpha_i & a_i \dot{q}_i \cos q_i \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \dot{q}_i =$$

$$= \begin{bmatrix} -\sin q_i & -\cos q_i \cos \alpha_i & \cos q_i \sin \alpha_i & -a_i \sin q_i \\ \cos q_i & -\sin q_i \cos \alpha_i & \sin q_i \sin \alpha_i & a_i \cos q_i \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \dot{q}_i$$

or in plain sight

$$\dot{A}_i = \Theta A_i \dot{q}_i$$

can be recorded.

If the kinematic couple is rotating,

$$\Theta = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

If the kinematic pair is related to the displacement, then the representation of $d_i = q_i = \text{var}$ and Θ is

$$\Theta = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

Using these matrices we have the following:

$$\frac{\partial \Gamma_i}{\partial q_j} = A_1 A_2 \dots A_{j-1} \Theta_j A_j \dots A_i, \quad 1 \leq j \leq i;$$

We define as follows

$$U_i^j = \frac{\partial \Gamma_i}{\partial q_j}.$$

The special derivatives with respect q_k ($j \leq k \leq i$) to the last singularity look like this:

$$U_i^{jk} = \frac{\partial U_i^j}{\partial q_k} = \frac{\partial^2 \Gamma_i}{\partial q_j \partial q_k} = A_1 A_2 \dots A_{j-1} \Theta_j A_j \dots A_{k-1} \Theta_k A_k \dots A_i, \quad 1 \leq j \leq i, j \leq k \leq i.$$

Using these kinematic equations, let's see the application of the K-controllability presented in the previous paragraph to a six-link Stanford robot [4]. A kinematic view of the Stanford robot is shown in Figure 1.

$O_0x_0y_0z_0$ - the coordinate system related to the base of the industrial robot ;

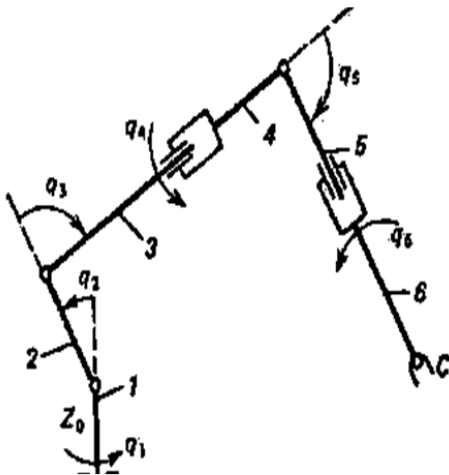
$O_i x_i y_i z_i$ ($1 < i < 5$) - coordinate systems related to the links of the industrial robot ;

$O_6 x_6 y_6 z_6$ - coordinate systems related to the grasping device of the industrial robot ;

θ_i - $O_{i-1} x_{i-1} y_{i-1} z_{i-1}$ -axis of the x_{i-1} coordinate system z_{i-1} quantity indicating the amount of rotation around the x_i axis until the axis is parallel to i . If the i -unit is a commutator, then d_i will be variable.

d_i - $O_{i-1} x_{i-1} y_{i-1} z_{i-1}$ in the coordinate system s , z_{i-1} along the axis i , x_i and z_{i-1} the distance to the point of intersection of the arrows.

α_i - x_i around the axis z_i arrow $i - z_{i-1}$ so that the axis is parallel to i a quantity indicating the angle of rotation.



1 - picture . Stanford of the robot kinematics appearance _

$$A_1 = \begin{pmatrix} \cos \theta_1 & -\cos \alpha_1 \sin \theta_1 & \sin \alpha_1 \sin \theta_1 & a_1 \cos \theta_1 \\ \sin \theta_1 & \cos \alpha_1 \cos \theta_1 & -\sin \alpha_1 \cos \theta_1 & a_1 \sin \theta_1 \\ 0 & \sin \alpha_1 & \cos \alpha_1 & d_1 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} \cos \theta_1 & 0 & -\sin \theta_1 & 0 \\ \sin \theta_1 & 0 & \cos \theta_1 & 0 \\ 0 & 0 & -1 & d_1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Var = d_1

$$A_2 = \begin{pmatrix} \cos \theta_2 & -\cos \alpha_2 \sin \theta_2 & \sin \alpha_2 \sin \theta_2 & a_2 \cos \theta_2 \\ \sin \theta_2 & \cos \alpha_2 \cos \theta_2 & -\sin \alpha_2 \cos \theta_2 & a_2 \sin \theta_2 \\ 0 & \sin \alpha_2 & \cos \alpha_2 & d_2 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} \cos \theta_2 & 0 & -\sin \theta_2 & 0 \\ \sin \theta_2 & 0 & \cos \theta_2 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Var = d_2

$$A_3 = \begin{pmatrix} \cos \theta_3 & -\cos \alpha_3 \sin \theta_3 & \sin \alpha_3 \sin \theta_3 & a_3 \cos \theta_3 \\ \sin \theta_3 & \cos \alpha_3 \cos \theta_3 & -\sin \alpha_3 \cos \theta_3 & a_3 \sin \theta_3 \\ 0 & \sin \alpha_3 & \cos \alpha_3 & d_3 \\ 0 & 0 & 0 & 1 \end{pmatrix} =$$

$$= \begin{pmatrix} \cos \theta_3 & 0 & -\sin \theta_3 & a_3 \cos \theta_3 \\ \sin \theta_3 & 0 & \cos \theta_3 & a_3 \sin \theta_3 \\ 0 & 1 & 0 & d_3 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\text{Var} = d_3$$

$$A_4 = \begin{pmatrix} \cos \theta_4 & -\cos \alpha_4 \sin \theta_4 & \sin \alpha_4 \sin \theta_4 & a_4 \cos \theta_4 \\ \sin \theta_4 & \cos \alpha_4 \cos \theta_4 & -\sin \alpha_4 \cos \theta_4 & a_4 \sin \theta_4 \\ 0 & \sin \alpha_4 & \cos \alpha_4 & d_4 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} \cos \theta_4 & 0 & -\sin \theta_4 & 0 \\ \sin \theta_4 & 0 & \cos \theta_4 & 0 \\ 0 & -1 & 0 & d_4 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$A_5 = \begin{pmatrix} \cos \theta_5 & -\cos \alpha_5 \sin \theta_5 & \sin \alpha_5 \sin \theta_5 & a_5 \cos \theta_5 \\ \sin \theta_5 & \cos \alpha_5 \cos \theta_5 & -\sin \alpha_5 \cos \theta_5 & a_5 \sin \theta_5 \\ 0 & \sin \alpha_5 & \cos \alpha_5 & d_5 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} \cos \theta_5 & 0 & -\sin \theta_5 & 0 \\ \sin \theta_5 & 0 & \cos \theta_5 & 0 \\ 0 & 1 & 0 & d_5 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$A_6 = \begin{pmatrix} \cos \theta_6 & -\cos \alpha_6 \sin \theta_6 & \sin \alpha_6 \sin \theta_6 & a_6 \cos \theta_6 \\ \sin \theta_6 & \cos \alpha_6 \cos \theta_6 & -\sin \alpha_6 \cos \theta_6 & a_6 \sin \theta_6 \\ 0 & \sin \alpha_6 & \cos \alpha_6 & d_6 \\ 0 & 0 & 0 & 1 \end{pmatrix} =$$

$$= \begin{pmatrix} \cos \theta_6 & -\sin \theta_6 & 0 & a_6 \cos \theta_6 \\ -\sin \theta_6 & \cos \theta_6 & 0 & a_6 \sin \theta_6 \\ 0 & 0 & 1 & d_6 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\text{Var} = d_6$$

$\theta_i, a_i, \alpha_i, d_i$ - we substitute the values of the parameters using the table 1 below.

1 - table . Stanford robot parameter values

Are you united ? the number i	θ_i	d_i	a_i	α_i
1	-90^0	d_1	0	-90^0
2	-90^0	d_2	0	90^0
3	-90^0	d_3	0	0^0
4	0^0	0	0	-90^0
5	0^0	0	0	90^0
6	0^0	d_6	0	0^0

H is for each resulting matrix, that is, for each element λ characteristics are derived from:
 the first unit:

$$\lambda_1 = 0,999, \quad \lambda_2 = 0,999, \quad \lambda_3 = 0,998, \quad \lambda_4 = 0,999.$$

Specific values for the second level:

$$\lambda_1 = 0,999, \quad \lambda_2 = 0,999, \quad \lambda_3 = 0,998, \quad \lambda_4 = 0,999.$$

Features for the third unit:

$$\lambda_1 = 0,997, \quad \lambda_2 = 0,998 \quad \lambda_3 = 0,997 \quad \lambda_4 = 0,998.$$

Specific values for the fourth unit:

$$\lambda_1 = 0,997, \quad \lambda_2 = 0,998 \quad \lambda_3 = 0,997 \quad \lambda_4 = 0,998.$$

the fifth unit:

$$\lambda_1 = 0,998, \quad \lambda_2 = 0,999, \quad \lambda_3 = 0,999, \quad \lambda_4 = 0,999.$$

Characteristics of the sixth unit :

$$\lambda_1 = 0,996, \quad \lambda_2 = 0,996, \quad \lambda_3 = 0,996, \quad \lambda_4 = 0,996.$$

matrices of these links are considered simultaneously, then it gives the motion matrix of the industrial robot 's working arm gripper . It is known that three operations are performed in the gripper of the Stanford robot. Depending on the importance of these operations, a decision is made as to how many control parameters can be used for the capture device . That is:

$$\Gamma = A_1 \cdot A_2 \cdot A_3 \cdot A_4 \cdot A_5 \cdot A_6$$

$$T_1 = A_1 \cdot A_2 \cdot A_3 = \begin{bmatrix} C_1 C_{23} & -S_1 & C_1 S_{23} & a_2 C_1 C_2 + a_3 C_1 C_{23} - d_2 S_1 \\ S_1 C_{23} & C_1 & S_1 S_{23} & a_2 S_1 C_2 + a_3 S_1 C_{23} + d_2 C_1 \\ -S_{23} & 0 & C_{23} & -a_2 S_2 - a_3 S_{23} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_2 = A_4 \cdot A_5 \cdot A_6 = \begin{bmatrix} C_4 C_5 C_6 - S_4 S_6 & -C_4 C_5 C_6 - S_4 S_6 & C_4 C_5 & d_6 C_4 C_5 \\ S_1 C_5 C_6 + C_4 S_6 & -S_4 C_5 C_6 + C_4 C_6 & S_4 S_5 & d_6 S_4 S_5 \\ -S_4 C_6 & S_5 S_6 & C_5 & d_6 C_5 + d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

here $C_i = \cos \theta_i, S_i = \sin \theta_i, C_{ij} = \cos(\theta_i + \theta_j), S_{ij} = \sin(\theta_i + \theta_j)$

$$A = T_1 \cdot T_2 = \begin{bmatrix} n_x & s_x & a_x & m_x \\ n_y & s_y & a_y & m_y \\ n_z & s_z & a_z & m_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

If the values of the variables are substituted using the table above, the following matrix will be obtained :

$$A = \begin{bmatrix} -1 & 0 & 0 & 0,40 \\ 0 & 0 & 1 & 0,65 \\ 0 & 1 & 0 & 0,70 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The following $\det[A - \lambda E] = 0$ if the determinant h is calculated, λ the characteristic equation depending on From the characteristic equation, $\lambda_1 = 0,999, \lambda_2 = 0,999, \lambda_3 = 0,999, \lambda_4 = 0,999$ comes from

As can be seen from the eigenvalues, they are multiples. It is not necessary to perform the remaining operations, such as the eigenvector of the system, the conversion of the matrix to Jordan normal form, because these are found based on the eigenvalues. Therefore, without further calculation of $P h$, if the matrix is written in Jordan normal form:

$$J(P) = \begin{bmatrix} \lambda_1 & 1 & 0 & 0 \\ 0 & \lambda_2 & 1 & 0 \\ 0 & 0 & \lambda_3 & 1 \\ 0 & 0 & 0 & \lambda_4 \end{bmatrix}$$

The subspace corresponding to each cell of Jordan's normal form:

$\lambda_1, \lambda_2, \lambda_3, \lambda_4$ – subspace corresponding to eigenvalues G_1 .

action space of SR G is, then $G = G_1$.

the matrix is selected (4×1) -based on the "K-controllability" method presented in the C – above paragraph. Then, to find out how well the received matrices are selected, they are checked according to the Kalman meson, $rank[(BC), A(BC), \dots, A^5(BC)] = 4$.

If the eigenvalues were different, then the size C – of the matrix (4×4) - would be the same. In general, C – it is in the form of a $(n \times k)$ - dimension of the matrix, and it means that the given system is k – controlled in a certain state. From this, it can be concluded that one control parameter will be necessary to control the industrial robot gripper.

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