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MUTUAL ONE-VALUE REFLECTION AND AUTOMORPHISMS

Aroyev Dilshod Davronovich
Kokand State Pedagogical Institute

Jumakulov Khurshidjon Kadiraliyevych
Kokand State Pedagogical Institute

Ummatova Makhbuba Akhmedovna
Kokand State Pedagogical Institute

Rafikov Fakhriddin Kakhkhorovich
Kokand State Pedagogical Institute

Annotation: this article presents how the mutual value reflections of all numerical systems on themselves look alike.

Keywords: reflection, put in place, attitude, reflections of a mutual value, ring.

In mathematics and its applications . A function is a special case of reflection. The definition and domain of a function are the same set, and it is easy to learn that the function is reciprocally one-valued. If the domain of definition and values of a function is a single set, which is finite, then the number of such functions and all their representations can be defined. If the number of elements of the set is n , then this set is called one-valued mappings to itself, and their number $n!$ is equal to.

The one-valued mappings of all number systems to themselves look like.

If the given set is infinite, and no condition is required for this set to be mutually equal-valued reflections, then the number of such reflections is obviously infinitely large. But if one or more algebraic structures (addition, multiplication of elements) are defined in the set, if it is required that the letters corresponding to the algebraic structures in the set are executed for the reciprocal one-valued reflections of this set , the number of such reflections may be finite or even unique.

We present the self-valued self-reflections of sets consisting of natural, integer, rational, real, complex numbers and quaternions.

n -natural numbers, that is, for arbitrary natural numbers $n+m\in N$ n and m , and $n\cdot m\in N$ the relations are fulfilled.

The set of natural, whole, rational, real, complex numbers by N,Z,Q,R,C letters, and the algebra of quaternions by A. In each of these sets, the operations + and · are defined .

Description. If the set K is one of the number systems listed above or an algebra of quaternions such that f:K→K is a one-valued reciprocal mapping

$$f(a+b)=f(a)+f(b) \tag{1}$$

$$f(a\cdot b)=f(a)\cdot f(b) \tag{2}$$

A reflection f is called an automorphism if the conditions hold for any elements. $a,b\in K$

For example $K=N$, using condition (2) of automorphism, we $f:N\to N$ can write equations for any $a\in N$ natural number . $f(a)=f(a\cdot 1)=f(a)\cdot f(1)$From the last equality and condition (1) of the automorphism

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$$f(2) = f(1+1) = f(1) + f(1) = 1+1 = 2$$

that is

$$f(2) = 2$$

we form the equation. Likewise

$$f(3) = 3,\ f(4) = 4, \ldots$$

equalities can be determined. From these $f : N \to N$, it can be determined that the automorphism consists of a unique reflection.

Let's say so $K = Z$. In this case, we also show that the $f : Z \to Z$ automorphism consists of an exact reflection, that is, the Z ring has only an automorphism consisting of an exact reflection . Using the condition (1) of the automorphism, the following equations can be written for an arbitrary a -integer:

$$f(a) = f(a+0) = f(a) + f(0) \text{ or } f(0) = 0$$

other integers f are only used to show transitions to themselves $f(1) = 1,\ f(-1) = -1$. The last equations derive from the following equations, respectively:

$$f(a) = f(a \cdot 1) = f(a) \cdot f(1) = a \cdot f(1),\ a \in Z$$

$$0 = f(1-1) = f(1+(-1)) = f(1) + f(-1) = 1 + f(-1)$$

Hence, the ring Z also has a unique automorphism.

Let $K = Q$ be an $f : Q \to Q$ arbitrary automorphism. As above, $f(0) = 0;\ f(1) = 1$ and any integer reflection f can be shown to be self-reflective. For this purpose, we show that any $\frac{1}{q}\ (q \in N)$ rational number f returns to itself by reflection.

$$1 = f(1) = f\left(q \cdot \frac{1}{q}\right) = f(q) \cdot f\left(\frac{1}{q}\right) = q \cdot f\left(\frac{1}{q}\right)$$

So $f\left(\frac{1}{q}\right) = \frac{1}{q}$ equality is appropriate. Using this

$$f\left(\frac{p}{q}\right) = f\left(p \cdot \frac{1}{q}\right) = f(p) \cdot f\left(\frac{1}{q}\right) = p \cdot \frac{1}{q}$$

we write the equation Thus, for any rational number $\frac{p}{q}$, $f\left(\frac{p}{q}\right) = \frac{p}{q}$ we have proved that the automorphism f is only a real permutation.

Suppose that $K = R$ the $f : R \to R$ mapping satisfies the automorphism conditions.

Theorem 1. The field of real numbers R has a unique automorphism consisting only of real permutations

.

Proof. Let be an $f: R \to R$ automorphism. $Q \subset R$ based on the relation and from the relation $f(0)=0,\ f(1)=1$ it can be proved as above that for $f\left(\frac{p}{q}\right)=\frac{p}{q}$ any rational number . $\frac{p}{q}$ Any irrational number for a complete proof of the theorem α It is sufficient to prove that f converges back to itself by reflection. First , we show that if is an α irrational number, it is $f(\alpha)$ also positive. since $\alpha > 0$ the number exists and the relationship $\sqrt{\alpha} \in R$ is $\sqrt{\alpha}$ appropriate. In that case,

$$f(\sqrt{\alpha}) = f(\sqrt{\alpha} \cdot \sqrt{\alpha}) = f(\sqrt{\alpha}) \cdot f(\sqrt{\alpha}) = \left(f(\sqrt{\alpha})\right)^2 > 0$$

attitude is appropriate. So it $f(\alpha) > 0$ will be. And it follows that if the relation $f(\alpha) < f(\beta)$ holds α for β real numbers , then the relation also holds $\alpha < \beta$. Hence, the automorphism f preserves the order relation in the field R. To complete the proof of the theorem, let us assume the converse, i.e. let the automorphism α f transfer an irrational number to an β irrational number and $f(\alpha) \neq \alpha$. since the $\alpha \neq \beta$ or, $\alpha < \beta$ or $\alpha > \beta$ relation is fulfilled. Suppose that the $\alpha < \beta$ inequality holds. There exists at least one rational r number between $\alpha < r < \beta$ any two irrational numbers in particular α and , that is, β. Under the assumption, the $\alpha < f(\alpha)$ inequality holds. Since the reflection f preserves the order relation

$$f(\alpha) < f(r) < f(\beta)$$

inequalities are valid or

$$\beta < f(r) < f(\beta)$$

But the $f(r) = r$ equality holds because r -is a rational number. Therefore $\beta < \alpha$, it holds, but the last $r < \beta$ inequality contradicts the inequality. So it $f(\alpha) = \alpha$ will be done. The theorem is proved.

Let be $K = C$ and be an $f: C \to C$ automorphism. In this case , the reflection f will not be unique, because the real reflection is another $f(\alpha) = \overline{\alpha}$, that is, the $f(a+bi) = a - bi$ reflection in the form is also an automorphism. In addition, the existence of infinitely many automorphisms of the field of complex numbers in [1] was shown without proof.

Below, we have the additional condition of automorphism, the field of complex numbers has only two, viz

$$f_1(a+bi) = a+bi$$
$$f_2(a+bi) = a-bi$$

to have automorphisms of the form

Theorem 2. If is an $f: C \to C$ automorphism and the $f(R) = R$ equality holds, that is, if any real number f can be converted back to a real number by reflection, then f_1 or f_2 is the same as reflection .

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Proof. is an automorphism such that the relation holds $f:C\to C$ for $f(a)\in R$ any real number $a\in R$. Then, using Theorem 1, we determine that the equality holds for any $a\in R$ real number . $f(a)=a$ If is an $\alpha=a+bi$ arbitrary complex number,

$$f(a+bi)=f(a)+f(b)f(i)=a+bf(i)$$
$$f(a-bi)=f(a)-f(b)f(i)=a-bf(i)$$

we write the equation. Of these

$$a^2+b^2=f((a+bi)(a+bi))=a^2-b^2(f(i))^2$$

equality follows. a and b considering that is an arbitrary real number

$$a^2+b^2=a^2-b^2(f(i))^2$$

equality $(f(i))^2=-1$ comes equality. $f(i)\in C$ from the relation and in the last equality $f(i)=i$ or $f(i)=-i$ we form the equality. Hence, only one of the equality , $f=f_1$, or $f_1=f_2$ is necessarily fulfilled. The theorem is proved.

Result: If is an $f:C\to C$ automorphism and is continuous, then $f=f_1$ either or , $f=f_2$ one of the equalities is definitely satisfied.

Proof. If there is an $f:C\to C$ automorphism, then it can be proved by the above method that any rational number goes back to itself using the reflection f . It is known that any irrational number can be expressed as the limit of a sequence of rational numbers. α and an arbitrary irrational number $\{r_n\}_1^\infty$ is α a sequence of rational numbers approximating to , i.e., $\lim_{n\to\infty} r_n=\alpha$ let f from the continuity of the automorphism the $f\left(\lim_{n\to\infty} r_n\right)=f(\alpha)$ equality is valid. Using this $\lim_{n\to\infty} f(r_n)=\lim_{n\to\infty} r_n$ equality,

$$f(\alpha)=f\left(\lim_{n\to\infty} r_n\right)=\lim_{n\to\infty} f(r_n)=\lim_{n\to\infty} r_n=\alpha$$

originates. So, every α real number is reflected back to itself using f reflection. From this and Theorem 2, it follows that one of $f=f_1$ the $f=f_2$ equalities is valid.

In order to continue the topic, we will introduce some concepts about the quaternion algebra, which is an extension of the field of complex numbers .

We accept the following definitions:

$$1=\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix};\ i=\begin{pmatrix}i & 0\\ 0 & i\end{pmatrix};\ j=\begin{pmatrix}0 & 1\\ -1 & 0\end{pmatrix};\ k=\begin{pmatrix}0 & i\\ i & 0\end{pmatrix}$$

It $i^2=j^2=k^2=-1$ can be proved that based on these definitions. These elements form the following table:

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		1		i
		k	1	
			i	1

Let's make the following set.

$$A = \{a + bi + cj + dk \mid a, b, c, d \in R\}$$

two $\alpha = a_1 + b_1 i + c_1 j + d_1 k$ and $\beta = a_2 + b_2 i + c_2 j + d_2 k$ elements in this set

$$\alpha + \beta = (a_1 + a_2) + (b_1 + b_2)i + (c_1 + c_2)j + (d_1 + d_2)k$$

their multiplication by k with equality is determined by the rule of multiplying a polynomial by a polynomial (in which i, j, k the multiplication of elements is explained by the table above), then the A set is a linear space over the field of real numbers. Also, the A set is a non-commutative ring. Such a set is called an algebra over the real numbers. This algebra is called the algebra of quaternions, and its elements are called quaternions. Denoting the algebra A of quaternions by , the $C \subset A$ relation becomes appropriate.

Any $\alpha \in A$ quaternion can be expressed in only one $\alpha = a + bi + cj + dk$ form. $\overline{\alpha} = a - bi - cj - dk$ A quaternion α is called a compound quaternion. It can be shown that α and $\overline{\alpha}$ for joint quaternions

$$\alpha \cdot \overline{\alpha} = a^2 + b^2 + c^2 + d^2 \in R \qquad (5)$$

attitude will be appropriate .

Now we determine what the automorphisms of $\alpha = a + bi + cj + dk$ quaternion algebra look like. A-be an arbitrary quaternion . In that case

$$f(\alpha) = f(a + bi + cj + dk) = f(a) + f(bi) + f(ci) + f(dk) = a + bf(i) + cf(j) + df(k)$$

and

$$f(\overline{\alpha}) = a - bf(i) - cf(j) - df(k)$$

equations can be formed. Of these

$$f(\alpha) \cdot f(\overline{\alpha}) = (a + bf(i) + cf(j) + df(k))(a - bf(i) - cf(j) - df(k))$$

or

$$f(\alpha \cdot \overline{\alpha}) = a^2 - b^2(f(i))^2 - c^2(f(j))^2 - d^2(f(k))^2$$

$\alpha \cdot \overline{\alpha} \in R$ and the reflection f leaves the actual number in place. that is why

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$$a^2+b^2+c^2+d^2=a^2-b^2(f(i))^2-c^2(f(j))^2-d^2(f(k))^2$$

a,b,c,d since s is an arbitrary real number

$$(f(i))^2=(f(j))^2=(f(k))^2=-1$$

we form the equation. Hence, $f(i), f(j), f(k)$s $i,-i, j,-j,k,-k$ is equal to one of the quaternions.

multiplying quaternions i, j and multiplying k elements. Therefore, the $f:A\rightarrow A$ reflection is an automorphism $f(\alpha\cdot\beta)=f(\alpha)\cdot f(\beta)$ instead of an equality to check the conditions

$$f(i,j)=f(i)\cdot f(j)$$
$$f(i,k)=f(i)\cdot f(k)$$
$$f(j,k)=f(j)\cdot f(k)$$

it will be enough to check that the equalities hold . As we have shown above, the $f(i), f(j)$ and $f(k)$ elements $i,-i, j,-j,k,-k$ are equal to one of the elements. A elements in algebra whose square is equal to -1 .

If $f:A\rightarrow A$ mirroring i, j, the elements are mirrored to the k elements and respectively $f(i), f(j), f(k)$

$$\begin{pmatrix} i & j & k \\ f(i) & f(j) & f(k) \end{pmatrix}$$

adding instead of ($f(i), f(j), f(k)$ elements i, j, k we assume that one of the elements is correspondingly equal) is odd, then f is not an automorphism. The odd substitutions are:

$$\varphi_1=\begin{pmatrix} i & j & k \\ j & i & k \end{pmatrix}, \varphi_2=\begin{pmatrix} i & j & k \\ k & j & i \end{pmatrix}, \varphi_3=\begin{pmatrix} i & j & k \\ i & k & j \end{pmatrix}$$

For example, φ_1 let's check the automorphism conditions for mapping to;

$$\varphi_1(i\cdot j)=\varphi_1(k)=k$$
$$\varphi_1(i)\cdot\varphi_1(j)=j\cdot k=-k \text{, but } k\neq -k$$
$$\varphi_1(i\cdot k)=\varphi_1(-j)=-\varphi_1(j)=i$$
$$\varphi_1(i)\cdot\varphi_1(k)=j\cdot k=j\cdot(i\cdot j)=-j\cdot(j\cdot i)=i$$
$$\varphi_1(k\cdot j)=\varphi_1(-i)=-\varphi_1(i)=j$$
$$\varphi_1(k)\cdot\varphi_1(j)=k\cdot i=(i\cdot j)\cdot i=-j\cdot i\cdot i=j$$

φ_2 and φ_3 it is also verified as above that the appropriate mapping is not an automorphism for substitutions. However, if we change one or three of the elements of the second row in the substitutions , φ_1, φ_2 and φ_3 to the opposite sign, the corresponding reflection is an automorphism. They are as follows:

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$$\varphi_4 = \begin{pmatrix} i & j & k \\ -j & i & k \end{pmatrix}, \varphi_5 = \begin{pmatrix} i & j & k \\ j & -i & k \end{pmatrix}, \varphi_6 = \begin{pmatrix} i & j & k \\ j & i & -k \end{pmatrix}, \varphi_7 = \begin{pmatrix} i & j & k \\ -j & -i & -k \end{pmatrix}$$

$$\varphi_8 = \begin{pmatrix} i & j & k \\ -k & j & i \end{pmatrix}, \varphi_9 = \begin{pmatrix} i & j & k \\ k & -j & i \end{pmatrix}, \varphi_{10} = \begin{pmatrix} i & j & k \\ k & j & -i \end{pmatrix}, \varphi_{11} = \begin{pmatrix} i & j & k \\ -k & -j & -i \end{pmatrix}$$

$$\varphi_{12} = \begin{pmatrix} i & j & k \\ -i & k & j \end{pmatrix}, \varphi_{13} = \begin{pmatrix} i & j & k \\ i & -k & j \end{pmatrix}, \varphi_{14} = \begin{pmatrix} i & j & k \\ i & k & -j \end{pmatrix}, \varphi_{15} = \begin{pmatrix} i & j & k \\ -i & -k & -j \end{pmatrix}$$

For example , a φ_4 quaternion is converted to an $a - bi - cj - dk$ element by means of a corresponding $f : A \rightarrow A$ reflection . $a + bi + cj + dk$ It should be remembered that we want to study only those A automorphisms satisfying the condition (a is an arbitrary real number) among the reciprocal one-valued reflections of the algebra . $f(a) = a$

$$\begin{pmatrix} i & j & k \\ f(i) & f(j) & f(k) \end{pmatrix}$$

three of the substitutions are even. They are as follows:

$$\varphi_{16} = \begin{pmatrix} i & j & k \\ i & j & k \end{pmatrix}, \varphi_{17} = \begin{pmatrix} i & j & k \\ j & k & i \end{pmatrix}, \varphi_{18} = \begin{pmatrix} i & j & k \\ k & i & j \end{pmatrix}$$

this pair of substitutions, their corresponding mapping satisfies the automorphism condition. We build them:

$$\varphi_{19} = \begin{pmatrix} i & j & k \\ -i & -j & k \end{pmatrix}, \varphi_{20} = \begin{pmatrix} i & j & k \\ -i & j & -k \end{pmatrix}, \varphi_{21} = \begin{pmatrix} i & j & k \\ i & -j & -k \end{pmatrix}$$

$$\varphi_{22} = \begin{pmatrix} i & j & k \\ -j & -k & i \end{pmatrix}, \varphi_{21} = \begin{pmatrix} i & j & k \\ -j & k & -i \end{pmatrix}, \varphi_{22} = \begin{pmatrix} i & j & k \\ j & -k & -i \end{pmatrix}$$

$$\varphi_{23} = \begin{pmatrix} i & j & k \\ -k & -i & j \end{pmatrix}, \varphi_{24} = \begin{pmatrix} i & j & k \\ -k & i & -j \end{pmatrix}, \varphi_{25} = \begin{pmatrix} i & j & k \\ k & -i & -j \end{pmatrix}$$

For example, φ_{25} a substitution reflection transforms a $a + bi + ci + di$ quaternion $a + bi - ci - di$ into a quaternion.

$$\varphi_{25}(i \cdot j) = \varphi_{25}(k) = -j$$

$$\varphi_{25}(i) \cdot \varphi_{25}(j) = k \cdot (-i) = -j$$

$$\varphi_{25}(i \cdot k) = \varphi_{25}(-j) = -\varphi_{25}(j) = -(-i) = i$$

$$\varphi_{25}(i) \cdot \varphi_{25}(k) = k \cdot (-j) = i \cdot j \cdot (-j) = -i \cdot j \cdot j = i$$

$$\varphi_{25}(k \cdot j) = \varphi_{25}(-i) = -\varphi_{25}(i) = -k$$

$$\varphi_{25}(k) \cdot \varphi_{25}(j) = -j \cdot (-i) = i \cdot j = -k$$

Thus, we Ahave shown that there are 25 automorphisms of algebra that transform real numbers into real numbers within their automorphisms.

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