

INTUITIONISTIC FUZZY π GENERALIZED SEMI PRE CONTINUOUS MAPPINGS

E.Dhanalakshmi¹, M.Vimala² & E.Rameshkumar³

¹Department of Mathematics, CMS College of Engineering and Technology, Tamilnadu, India

²Department of Mathematics, CMS College of Engineering and Technology, Tamilnadu, India

³Department of Mathematics, CMS College of Engineering and Technology, Tamilnadu, India

Email: rameshfuzzy@gmail.com, dhanaharshad@gmail.com

Abstract : *In this paper we have introduced intuitionistic fuzzy π generalized semi precontinuous mappings and studied some of their basic properties.*

Key words : *Intuitionistic fuzzy topology, intuitionistic fuzzy π generalized semi pre closed sets, intuitionistic fuzzy π generalized semi continuous mappings, intuitionistic fuzzy $T_{1/2}$ (IFT_{1/2}) space and intuitionistic fuzzy SPT_{1/2} (IFSP T_{1/2}) space.*

1. Introduction

The concept of fuzzy sets was introduced by Zadeh [13] and later Atanassov [1] generalized this idea to intuitionistic fuzzy sets using the notion of fuzzy sets. On the other hand Coker [4] introduced intuitionistic fuzzy topological spaces using the notion of intuitionistic fuzzy sets. In this paper, we introduced intuitionistic fuzzy π generalized semi pre continuous mappings and studied some of their basic properties. We arrived at some characterizations of intuitionistic fuzzy π generalized semi pre continuous mappings.

II. Preliminaries

Definition 2.1: [1] Let X be a non empty fixed set. An intuitionistic fuzzy set (IFS in short) A in X is an object having the form

$$A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle / x \in X \}$$

where the functions $\mu_A(x): X \rightarrow [0, 1]$ and $\nu_A(x): X \rightarrow [0, 1]$ denote the degree of membership (namely $\mu_A(x)$) and the degree of non-membership (namely $\nu_A(x)$) of each element $x \in X$ to the set A , respectively, and $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ for each $x \in X$. Denote the set of all intuitionistic fuzzy sets in X by IFS (X).

Definition 2.2: [1] Let A and B be IFSs of the form

$$A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle / x \in X \} \text{ and } B = \{ \langle x, \mu_B(x), \nu_B(x) \rangle / x \in X \}.$$

(a) $A \subseteq B$ if and only if $\mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x)$ for all $x \in X$

(b) $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$

(c) $A^c = \{ \langle x, \nu_A(x), \mu_A(x) \rangle / x \in X \}$

(d) $A \cap B = \{ \langle x, \mu_A(x) \wedge \mu_B(x), \nu_A(x) \vee \nu_B(x) \rangle / x \in X \}$

(e) $A \cup B = \{ \langle x, \mu_A(x) \vee \mu_B(x), \nu_A(x) \wedge \nu_B(x) \rangle / x \in X \}$

For the sake of simplicity, we shall use the notation $A = \langle x, \mu_A, \nu_A \rangle$ instead of $A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle / x \in X \}$. Also for the sake of simplicity, we shall use the notation $A = \{ \langle x, (\mu_A, \mu_B), (\nu_A, \nu_B) \rangle \}$ instead of $A = \langle x, (A/\mu_A, B/\mu_B), (A/\nu_A, B/\nu_B) \rangle$.

The intuitionistic fuzzy sets $0_- = \{ \langle x, 0, 1 \rangle / x \in X \}$ and $1_- = \{ \langle x, 1, 0 \rangle / x \in X \}$ are respectively the empty set and the whole set of X .

Definition 2.3: [3] An intuitionistic fuzzy topology (IFT in short) on X is a family τ of IFSs in X satisfying the following axioms.

(i) $0_-, 1_- \in \tau$

(ii) $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$

(iii) $\cup G_i \in \tau$ for any family $\{ G_i / i \in J \} \subseteq \tau$.

In this case the pair (X, τ) is called an intuitionistic fuzzy topological space (IFTS in short) and any IFS in τ is known as an intuitionistic fuzzy open set (IFOS in short) in X .

The complement A^c of an IFOS A in IFTS (X, τ) is called an intuitionistic fuzzy closed set (IFCS in short) in X .

Definition 2.4:[3] Let (X, τ) be an IFTS and $A = \langle x, \mu_A, \nu_A \rangle$ be an IFS in X . Then the intuitionistic fuzzy interior and intuitionistic fuzzy closure are defined by

$$\text{int}(A) = \cup \{ G / G \text{ is an IFOS in } X \text{ and } G \subseteq A \},$$

$$\text{cl}(A) = \cap \{ K / K \text{ is an IFCS in } X \text{ and } A \subseteq K \}.$$

Note that for any IFS A in (X, τ) , we have $\text{cl}(A^c) = [\text{int}(A)]^c$ and $\text{int}(A^c) = [\text{cl}(A)]^c$.

Definition 2.5: [4] An IFS $A = \langle x, \mu_A, \nu_A \rangle$ in an IFTS (X, τ) is said to be a

(i) intuitionistic fuzzy semi closed set (IFSCS for short) if $\text{int}(\text{cl}(A)) \subseteq A$,

(ii) intuitionistic fuzzy pre-closed set (IFPCS for short) if $\text{cl}(\text{int}(A)) \subseteq A$,

(iii) intuitionistic fuzzy α -closed set (IF α CS for short) if $\text{cl}(\text{int}(\text{cl}(A))) \subseteq A$,

(iv) intuitionistic fuzzy β -closed set (IF β CS for short) if $\text{int}(\text{cl}(\text{int}(A))) \subseteq A$.

The respective complements of the above IFCSs are called their respective IFOSs.

The family of all IFSCSs, IFPCSs, IF α CSs and IF β CSs (respectively IFOSOs, IFPOOs, IF α OOs and IF β OOs) of an IFTS (X, τ) are respectively denoted by IFSC(X), IFPC(X), IF α C(X) and IF β C(X) (respectively IFSO(X), IFPO(X), IF α O(X) and IF β O(X)).

Definition 2.6:[12] Let A be an IFS in an IFTS (X, τ) . Then

$$\text{sint}(A) = \cup \{ G / G \text{ is an IFOS in } X \text{ and } G \subseteq A \},$$

$$\text{scl}(A) = \cap \{ K / K \text{ is an IFSCS in } X \text{ and } A \subseteq K \}.$$

Note that for any IFS A in (X, τ) , we have $\text{scl}(A^c) = (\text{sint}(A))^c$ and $\text{sint}(A^c) = (\text{scl}(A))^c$.

Definition 2.7:[11] An IFS A in an IFTS (X, τ) is an

(i) intuitionistic fuzzy generalized closed set (IFGCS in short) if $\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFOS in X .

Definition 2.8:[10] An IFS A in an IFTS (X, τ) is said to be an intuitionistic fuzzy generalized semi closed set (IFGSCS in short) if $\text{scl}(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFOS in (X, τ) .

Definition 2.9:[10] An IFS A is said to be an intuitionistic fuzzy generalized semi open set (IFGSOS in short) in X if the complement A^c is an IFGSCS in X .

The family of all IFGSCSs (IFGSOSs) of an IFTS (X, τ) is denoted by IFGSC(X) (IFGSO(X)).

Definition 2.10:[8] An IFS A in an IFTS (X, τ) is said to be an intuitionistic fuzzy π -generalized semi closed set (IF π GSCS in short) if $\text{scl}(A) \subseteq U$ whenever $A \subseteq U$ and U is an IF π OS in (X, τ) .

Definition 2.11:[5] Let f be a mapping from an IFTS (X, τ) into an IFTS (Y, σ) . Then f is said to be intuitionistic fuzzy continuous (IF continuous in short) if $f^{-1}(B) \in \text{IFO}(X)$ for every $B \in \sigma$.

Definition 2.12: [5] Let f be a mapping from an IFTS (X, τ) into an IFTS (Y, σ) . Then f is said to be an

(i) intuitionistic fuzzy semi continuous mapping (IFS continuous mapping for short) if $f^{-1}(B) \in \text{IFSO}(X)$ for every $B \in \sigma$

(ii) intuitionistic fuzzy α -continuous mapping (IF α continuous mapping for short) if $f^{-1}(B) \in \text{IF}\alpha\text{O}(X)$ for every $B \in \sigma$

(iii) intuitionistic fuzzy pre continuous mapping (IFP continuous mapping for short) if $f^{-1}(B) \in \text{IFPO}(X)$ for every $B \in \sigma$

- (iv) intuitionistic fuzzy β continuous mapping (IF β continuous mapping for short) if $f^{-1}(B) \in \text{IF}\beta\text{O}(X)$ for every $B \in \sigma$.

Definition 2.13: [12] Let f be a mapping from an IFTS (X, τ) into an IFTS (Y, σ) . Then f is said to be an intuitionistic fuzzy generalized continuous mapping (IFG continuous mapping for short) if $f^{-1}(B) \in \text{IFGC}(X)$ for every IFCS B in Y .

Definition 2.14: [12] Let f be a mapping from an IFTS (X, τ) into an IFTS (Y, σ) . Then f is said to be an intuitionistic fuzzy semi-pre continuous mapping (IFSP continuous mapping for short) if $f^{-1}(B) \in \text{IFSP}\text{O}(X)$ for every $B \in \sigma$.

Result 2.15:[10] Every IF continuous mapping is an IFG continuous mapping.

Definition 2.16:[9] A mapping $f: (X, \tau) \rightarrow (Y, \sigma)$ is called an *intuitionistic fuzzy generalized semi continuous* (IFGS continuous in short) if $f^{-1}(B)$ is an IFGSCS in (X, τ) for every IFCS B of (Y, σ) .

Definition 2.17: [11] An IFTS (X, τ) is said to be an intuitionistic fuzzy $T_{1/2}$ space (IFT $_{1/2}$ space for short) if every IFGSCS in (X, τ) is an IFCS in (X, τ) .

Definition 2.18: [9] If every IF π GSPCS in (X, τ) is an IFSPCS in (X, τ) , then the space can be called as an intuitionistic fuzzy semi-pre $T_{1/2}$ space (IFSPT $_{1/2}$ for short).

Definition 2.19: [10] Let $c(\alpha, \beta)$ be an IFP of an IFTS (X, τ) . An IFS A of X is called an intuitionistic fuzzy neighborhood (IFN for short) of $c(\alpha, \beta)$ if there exists an IFOS B in X such that $c(\alpha, \beta) \in B \subseteq A$.

Theorem 2.20 : [9] For any IFS A in (X, τ) where X is an IFSPT $_{1/2}$ space, $A \in \text{IF}\pi\text{GSPO}(X)$ if and only if for every IFP $c(\alpha, \beta) \in A$, there exists an IF π GSPOS in X such that $c(\alpha, \beta) \in B \subseteq A$.

III. Intuitionistic fuzzy π generalized semi-pre continuous mappings

In this section we have introduced intuitionistic fuzzy π generalized semi-pre continuous mappings and investigated some of their properties.

Definition 3.1: A mapping $f: (X, \tau) \rightarrow (Y, \sigma)$ is called an intuitionistic fuzzy π generalized semi-pre continuous (IF π GSP continuous for short) mapping if $f^{-1}(B)$ is an IF π GSPCS in (X, τ) for every IFCS B of (Y, σ) .

Example 3.2: Let $X = \{a, b\}$, $Y = \{u, v\}$ and $G_1 = \langle x, (0.4, 0.3), (0.4, 0.5) \rangle$, $G_2 = \langle y, (0.5, 0.6), (0.3, 0.1) \rangle$. Then $\tau = \{0_-, G_1, 1_-\}$ and $\sigma = \{0_-, G_2, 1_-\}$ are IFTs on X and Y respectively. Define a mapping $f: (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. Then f is an IF π GSP continuous mapping.

Proposition 3.3: If $f: (X, \tau) \rightarrow (Y, \sigma)$ is an IF continuous mapping then it is an IF π GSP continuous mapping.

Proof: Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be an IF continuous mapping. Let A be an IFCS in (Y, σ) . Since f is IF continuous mapping, $f^{-1}(A)$ is an IFCS in (X, τ) . Since every IFCS is an IF π GSPCS, $f^{-1}(A)$ is an IF π GSPCS in (X, τ) . Hence f is an IF π GSP continuous mapping.

The converse of the above proposition need not be true as seen from the following example.

Example 3.4: Let $X = \{a, b\}$, $Y = \{u, v\}$ and $G_1 = \langle x, (0.2, 0.2), (0.7, 0.5) \rangle$, $G_2 = \langle y, (0.7, 0.5), (0.2, 0.2) \rangle$. Then $\tau = \{0_-, G_1, 1_-\}$ and $\sigma = \{0_-, G_2, 1_-\}$ are IFTs on X and Y respectively. Define a mapping $f: (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. The IFS $A = \langle y, (0.2, 0.2), (0.7, 0.5) \rangle$ is IFCS in (Y, σ) . Then $f^{-1}(A)$ is

$IF\pi GSPCS$ in (X, τ) but not $IFCS$ in (X, τ) . Therefore, f is an $IF\pi GSP$ continuous mapping but not an IF continuous mapping.

Proposition 3.5: Every IFG continuous mapping is an $IF\pi GSP$ continuous mapping but not conversely.

Proof: Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be an IFG continuous mapping. Let A be an $IFCS$ in (Y, σ) . Since f is an IFG continuous mapping, $f^{-1}(A)$ is an $IFGCS$ in (X, τ) . Since every $IFGCS$ is an $IF\pi GSPCS$, $f^{-1}(A)$ is an $IF\pi GSPCS$ in (X, τ) . Hence f is an $IF\pi GSP$ continuous mapping.

The converse of the above proposition need not be true as seen from the following example.

Example 3.6: Let $X = \{ a, b \}$, $Y = \{ u, v \}$ and $G_1 = \langle x, (0.3, 0.2), (0.4, 0.5) \rangle$, $G_2 = \langle y, (0.4, 0.5), (0.3, 0.2) \rangle$. Then $\tau = \{0_-, G_{1,1_-}\}$ and $\sigma = \{0_-, G_{2,1_-}\}$ are IFTs on X and Y respectively. Define a mapping $f: (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. The IFS $A = \langle y, (0.3, 0.2), (0.4, 0.5) \rangle$ is an $IFCS$ in (Y, σ) . Then $f^{-1}(A)$ is an $IF\pi GSPCS$ in (X, τ) but not an $IFGCS$ in (X, τ) . Therefore, f is an $IF\pi GSP$ continuous mapping but not an IFG continuous mapping.

Proposition 3.7: If $f: (X, \tau) \rightarrow (Y, \sigma)$ is an $IF\alpha$ continuous mapping then it is an $IF\pi GSP$ continuous mapping.

Proof: Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be an $IF\alpha$ continuous mapping. Let A be an $IFCS$ in (Y, σ) . Since f is an $IF\alpha$ continuous mapping, $f^{-1}(A)$ is an $IF\alpha CS$ in (X, τ) . Since every $IF\alpha CS$ is an $IF\pi GSPCS$, $f^{-1}(A)$ is an $IF\pi GSPCS$ in (X, τ) . Hence f is an $IF\pi GSP$ continuous mapping.

The converse of the above proposition need not be true as seen from the following example.

Example 3.8: Let $X = \{ a, b \}$, $Y = \{ u, v \}$ and $G_1 = \langle x, (0.6, 0.7), (0.2, 0.1) \rangle$, $G_2 = \langle y, (0.1, 0.2), (0.4, 0.3) \rangle$. Then $\tau = \{0_-, G_{1,1_-}\}$ and $\sigma = \{0_-, G_{2,1_-}\}$ are IFTs on X and Y respectively. Define a mapping $f: (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. The IFS $A = \langle y, (0.4, 0.3), (0.1, 0.2) \rangle$ is an $IFCS$ in (Y, σ) . Then $f^{-1}(A)$ is an $IF\pi GSPCS$ in (X, τ) but not an $IF\alpha CS$ in (X, τ) . Therefore, f is an $IF\pi GSP$ continuous mapping but not an $IF\alpha$ continuous mapping.

Proposition 3.9: Every IFR continuous mapping is an $IF\pi GSP$ continuous mapping but not conversely.

Proof: Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be an IFR continuous mapping. Let A be an $IFCS$ in (Y, σ) . Then by hypothesis $f^{-1}(A)$ is an $IFRCS$ in (X, τ) . Since every $IFRCS$ is an $IF\pi GSPCS$, $f^{-1}(A)$ is an $IF\pi GSPCS$ in (X, τ) . Hence f is an $IF\pi GSP$ continuous mapping.

Example 3.10: Let $X = \{ a, b \}$, $Y = \{ u, v \}$ and $G_1 = \langle x, (0.2, 0.3), (0.4, 0.5) \rangle$, $G_2 = \langle y, (0.4, 0.5), (0.2, 0.3) \rangle$. Then $\tau = \{0_-, G_{1,1_-}\}$ and $\sigma = \{0_-, G_{2,1_-}\}$ are IFTs on X and Y respectively. Define a mapping $f: (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. The IFS $A = \langle y, (0.2, 0.3), (0.4, 0.5) \rangle$ is an $IFCS$ in (Y, σ) . Then $f^{-1}(A)$ is $IF\pi GSPCS$ in (X, τ) but not an $IFRCS$ in (X, τ) . Therefore, f is an $IF\pi GSP$ continuous mapping but not an IFR continuous mapping.

Proposition 3.11: Every IFS continuous mapping is an $IF\pi GSP$ continuous mapping but not conversely.

Proof: Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be an IFS continuous mapping. Let A be an $IFCS$ in (Y, σ) . Since f is an IFS continuous mapping, $f^{-1}(A)$ is an $IFSCS$ in (X, τ) . Since every $IFSCS$ is an $IF\pi GSPCS$, $f^{-1}(A)$ is an $IF\pi GSPCS$ in (X, τ) . Hence f is an $IF\pi GSP$ continuous mapping.

Example 3.12: Let $X = \{ a, b \}$, $Y = \{ u, v \}$ and $G_1 = \langle x, (0.1, 0.2), (0.4, 0.6) \rangle$, $G_2 = \langle y, (0.5, 0.7), (0.0, 0.1) \rangle$. Then $\tau = \{0_-, G_{1,1_-}\}$ and $\sigma = \{0_-, G_{2,1_-}\}$ are IFTs on X and Y respectively. Define a mapping $f: (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. The IFS $A = \langle y, (0.0, 0.1), (0.5, 0.7) \rangle$ is an $IFCS$ in (Y, σ) . Then $f^{-1}(A)$ is $IF\pi GSPCS$ in (X, τ) but not an $IFSCS$ in (X, τ) . Therefore, f is an $IF\pi GSP$ continuous mapping but not an IFS continuous mapping.

Proposition 3.13: Every IFP continuous mapping is an $IF\pi GSP$ continuous mapping but not conversely.

Theorem 3.20: If $f: (X, \tau) \rightarrow (Y, \sigma)$ is an IF π GSP continuous mapping then for each IFP $c(\alpha, \beta)$ of X and each $A \in \sigma$ such that $f(c(\alpha, \beta)) \subseteq A$, there exists an IF π GSPOS B of X such that $c(\alpha, \beta) \subseteq B$ and $f(B) \subseteq A$.

Proof: Let $c(\alpha, \beta)$ be an IFP of X and $A \in \sigma$ such that $f(c(\alpha, \beta)) \subseteq A$. Put $B = f^{-1}(A)$. Then by hypothesis B is an IF π GSPOS in X such that $c(\alpha, \beta) \subseteq B$ and $f(B) = f(f^{-1}(A)) \subseteq A$.

Theorem 3.21: Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be an IF π GSP continuous mapping, then f is an IFSP continuous mapping if X is an IFSPT $_{1/2}$ space.

Proof: Let V be an IFCS in Y . Then $f^{-1}(V)$ is an IF π GSPCS in X , by hypothesis. Since X is an IFSPT $_{1/2}$ space, $f^{-1}(V)$ is an IFSPCS in X . Hence f is an IFSP continuous mapping.

Theorem 3.22: Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be an IF π GSP continuous mapping and $g: (Y, \sigma) \rightarrow (Z, \gamma)$ is an IFG continuous mapping and Y is an IFT $_{1/2}$ space, then $g \circ f: (X, \tau) \rightarrow (Z, \gamma)$ is an IF π GSP continuous mapping.

Proof: Let V be an IFCS in Z . Then $g^{-1}(V)$ is an IFGCS in Y , by hypothesis. Since Y is an IFT $_{1/2}$ space, $g^{-1}(V)$ is an IFCS in Y . Therefore $f^{-1}(g^{-1}(V))$ is an IF π GSPCS in X , by hypothesis. Hence $g \circ f$ is an IF π GSP continuous mapping.

Theorem 3.23: Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be an IF π GSP continuous mapping and $g: (Y, \sigma) \rightarrow (Z, \gamma)$ is an IF continuous mapping, then $g \circ f: (X, \tau) \rightarrow (Z, \gamma)$ is an IF π GSP continuous mapping.

Proof: Let V be an IFCS in Z . Then $g^{-1}(V)$ is an IFCS in Y , by hypothesis. Since f is an IF π GSP continuous mapping, $f^{-1}(g^{-1}(V))$ is an IF π GSPCS in X . Hence $g \circ f$ is an IF π GSP continuous mapping.

Theorem 3.24: Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be a mapping from an IFTS X into an IFTS Y . Then the following conditions are equivalent if X and Y are IFSPT $_{1/2}$ spaces.

- (i) f is an IF π GSP continuous mapping
- (ii) $f^{-1}(B)$ is an IF π GSPOS in X for each IFOS B in Y
- (iii) for every IFP $c(\alpha, \beta)$ in X and for every IFOS B in Y such that $f(c(\alpha, \beta)) \in B$, there exists an IF π GSPOS A in X such that $c(\alpha, \beta) \in A$ and $f(A) \subseteq B$.

Proof: (i) \Leftrightarrow (ii) is obvious, since $f^{-1}(A^c) = (f^{-1}(A))^c$.

(ii) \Rightarrow (iii) Let B be any IFOS in Y and let $c(\alpha, \beta) \in X$. Given $f(c(\alpha, \beta)) \in B$. By hypothesis $f^{-1}(B)$ is an IF π GSPOS in X . Take $A = f^{-1}(B)$. Now $c(\alpha, \beta) \in f^{-1}(f(c(\alpha, \beta)))$. Therefore $f^{-1}(f(c(\alpha, \beta))) \in f^{-1}(B) = A$. This implies $c(\alpha, \beta) \in A$ and $f(A) = f(f^{-1}(B)) \subseteq B$.

(iii) \Rightarrow (i) Let A be an IFCS in Y . Then its complement, say B is an IFOS in Y . Let $c(\alpha, \beta) \in X$ and $f(c(\alpha, \beta)) \in B$. Then there exists an IF π GSPOS, say $C = f^{-1}(B)$ in X such that $c(\alpha, \beta) \in C$ and $f(C) \subseteq B$. Therefore $f^{-1}(B)$ is an IF π GSPOS in X , by Theorem 2.20. That is $f^{-1}(A^c)$ is an IF π GSPOS in X and hence $f^{-1}(A)$ is an IF π GSPCS in X . Thus f is an IF π GSP continuous mapping.

Theorem 3.25: Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be a mapping from an IFTS X into an IFTS Y . Then the following conditions are equivalent if X and Y are IFSPT $_{1/2}$ spaces.

- (i) f is an IF π GSP continuous mapping
- (ii) for each IFP $c(\alpha, \beta)$ in X and for every IFN A of $f(c(\alpha, \beta))$, there exists an IFGSPSPOS B in X such that $c(\alpha, \beta) \in B \subseteq f^{-1}(A)$.
- (iii) for each IFP $c(\alpha, \beta)$ in X and for every IFN A of $f(c(\alpha, \beta))$, there exists an IF π GSPOS B in X such that $c(\alpha, \beta) \in B$ and $f(B) \subseteq A$.

Proof: (i) \Rightarrow (ii) Let $c(\alpha, \beta) \in X$ and let A be an IFN of $f(c(\alpha, \beta))$. Then there exists an IFOS C in Y such that $f(c(\alpha, \beta)) \in C \subseteq A$. Since f is an IF π GSP continuous mapping, $f^{-1}(C) = B$ (say), is an IF π GSPOS in X and $c(\alpha, \beta) \in B \subseteq f^{-1}(A)$.

(ii) \Rightarrow (iii) Let $c(\alpha, \beta) \in X$ and let A be an IFN of $f(c(\alpha, \beta))$. Then there exists an IF π GSPOS B in X such that $c(\alpha, \beta) \in B \subseteq f^{-1}(A)$, by hypothesis. Therefore, $c(\alpha, \beta) \in B$ and $f(B) \subseteq f(f^{-1}(A)) \subseteq A$.

(iii) \Rightarrow (i) Let B be an IF π OS in Y and let $c(\alpha, \beta) \in f^{-1}(B)$. Then $f(c(\alpha, \beta)) \in B$. Therefore B is an IFN of $f(c(\alpha, \beta))$. Since B is IF π OS, by hypothesis there exists an IF π GSPOS A in X such that $c(\alpha, \beta) \in A \subseteq f^{-1}(f(A)) \subseteq f^{-1}(B)$. Therefore, $f^{-1}(B)$ is an IF π GSPOS in X , by Theorem 2.20. Hence f is an IF π GSP continuous mapping.

Theorem 3.26: Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be a mapping from an IFTS X into an IFTS Y . Then the following conditions are equivalent if X is an IFSPT $_{1/2}$ space.

- (i) f is an IF π GSP continuous mapping
- (ii) If B is an IFOS in Y then $f^{-1}(B)$ is an IF π GSPOS in X
- (iii) $f^{-1}(\text{int}(B)) \subseteq \text{cl}(\text{int}(\text{cl}(f^{-1}(B))))$ for every IFS B in Y .

Proof: (i) \Leftrightarrow (ii) is obviously true by theorem 3.24

(ii) \Rightarrow (iii) Let B be any IFS in Y . Then $\text{int}(B)$ is an IFOS in Y . Then $f^{-1}(\text{int}(B))$ is an IF π GSPOS in X . Since X is an IFSPT $_{1/2}$ space, $f^{-1}(\text{int}(B))$ is an IFSPOS in X . Therefore, $f^{-1}(\text{int}(B)) \subseteq \text{cl}(\text{int}(\text{cl}(f^{-1}(\text{int}(B)))) \subseteq \text{cl}(\text{int}(\text{cl}(f^{-1}(B))))$.

(iii) \Rightarrow (i) Let B be an IFOS in Y . By hypothesis $f^{-1}(B) = f^{-1}(\text{int}(B)) \subseteq \text{cl}(\text{int}(\text{cl}(f^{-1}(B))))$. This implies $f^{-1}(B)$ is an IF β OS in X . Therefore, $f^{-1}(B)$ is an IF π GSPOS in X and hence by theorem 3.24, f is an IF π GSP continuous mapping.

Theorem 3.27: Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be a mapping from an IFTS X into an IFTS Y . Then the following conditions are equivalent if X and Y are IFSPT $_{1/2}$ spaces.

- (i) f is an IF π GSP continuous mapping
- (ii) $\text{int}(\text{cl}(\text{int}(f^{-1}(B)))) \subseteq f^{-1}(\text{spcl}(B))$ for each IFCS B in Y
- (iii) $f^{-1}(\text{spint}(B)) \subseteq \text{cl}(\text{int}(\text{cl}(f^{-1}(B))))$ for each IFOS B of Y
- (iv) $f(\text{int}(\text{cl}(\text{int}(A)))) \subseteq \text{cl}(f(A))$ for each IFS A of X .

Proof: (i) \Rightarrow (ii) Let B be an IFCS in Y . Then $f^{-1}(B)$ is an IF π GSPCS in X . Since X is IFSPT $_{1/2}$ space, $f^{-1}(B)$ is an IFSPCS. Therefore, $\text{int}(\text{cl}(\text{int}(f^{-1}(B)))) \subseteq f^{-1}(B) = f^{-1}(\text{spcl}(B))$

(ii) \Rightarrow (iii) can be easily proved by taking complement in (ii).

(iii) \Rightarrow (iv) Let $A \in X$. Then $B = f(A)$ and therefore, $A \subseteq f^{-1}(B)$. Here $\text{int}(f(A)) = \text{int}(B)$ is an IFOS in Y . Then (iii) implies that, $f^{-1}(\text{spint}(\text{int}(B))) \subseteq \text{cl}(\text{int}(\text{cl}(f^{-1}(\text{int}(B)))) \subseteq \text{cl}(\text{int}(\text{cl}(f^{-1}(B))))$. Now $(\text{cl}(\text{int}(\text{cl}(A^c))))^c \subseteq (\text{cl}(\text{int}(\text{cl}(f^{-1}(B^c))))^c \subseteq (f^{-1}(\text{spint}(\text{int}(B^c))))^c$. Therefore $\text{int}(\text{cl}(\text{int}(A))) \subseteq f^{-1}(\text{spcl}(\text{cl}(B)))$. Now $f(\text{int}(\text{cl}(\text{int}(A)))) \subseteq f(f^{-1}(\text{spcl}(\text{cl}(B)))) \subseteq \text{cl}(B) = \text{cl}(f(A))$.

(iv) \Rightarrow (i) Let B be any IFCS in Y , then $f^{-1}(B)$ is an IFS in X . By hypothesis $f(\text{int}(\text{cl}(\text{int}(f^{-1}(B)))) \subseteq \text{cl}(f(f^{-1}(B))) \subseteq \text{cl}(B) = B$. Now $\text{int}(\text{cl}(\text{int}(f^{-1}(B)))) \subseteq f^{-1}(f(\text{int}(\text{cl}(\text{int}(f^{-1}(B)))) \subseteq f^{-1}(B)$. This implies, $f^{-1}(B)$ is an IF β CS and hence $f^{-1}(B)$ is an IF π GSPCS in X . Thus f is an IF π GSP continuous mapping.

Theorem 3.28: A mapping $f: (X, \tau) \rightarrow (Y, \sigma)$ is an IF π GSP continuous mapping if $\text{cl}(\text{int}(\text{cl}(f^{-1}(A)))) \subseteq f^{-1}(\text{cl}(A))$ for every IFS A in Y .

Proof: Let A be an IFOS in Y then A^c is an IFCS in Y . By hypothesis, $\text{cl}(\text{int}(\text{cl}(f^{-1}(A^c)))) \subseteq f^{-1}(\text{cl}(A^c)) = f^{-1}(A^c)$, since A^c is an IFCS. Now $(\text{int}(\text{cl}(\text{int}(f^{-1}(A))))^c = \text{cl}(\text{int}(\text{cl}(f^{-1}(A^c)))) \subseteq f^{-1}(A^c) = (f^{-1}(A))^c$. This implies $f^{-1}(A) \subseteq \text{int}(\text{cl}(\text{int}(f^{-1}(A))))$. Hence $f^{-1}(A)$ is an IF α OS in X and hence it is an IF π GSPOS in X . Therefore, by theorem 3.24, f is an IF π GSP continuous mapping.

Theorem 3.29: Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be a mapping from an IFTS X into an IFTS Y . Then the following conditions are equivalent if X is an IFSPT $_{1/2}$ space.

- (i) f is an IF π GSP continuous mapping
- (ii) $f^{-1}(B)$ is an IF π GSPCS in X for every IFCS B in Y
- (iii) $\text{int}(\text{cl}(\text{int}(f^{-1}(A)))) \subseteq f^{-1}(\text{cl}(A))$ for every IFS A in Y .

Proof: (i) \Leftrightarrow (ii) is obvious from the Definition 3.1.

(ii) \Rightarrow (iii) Let A be an IFS in Y . Then $\text{cl}(A)$ is an IFCS in Y . By hypothesis, $f^{-1}(\text{cl}(A))$ is an IF π GSPCS in X . Since X is an IFSPT $_{1/2}$ space, $f^{-1}(\text{cl}(A))$ is an IFSPCS. Therefore $\text{int}(\text{cl}(\text{int}(f^{-1}(\text{cl}(A)))) \subseteq f^{-1}(\text{cl}(A))$. Now $\text{int}(\text{cl}(\text{int}(f^{-1}(A)))) \subseteq \text{int}(\text{cl}(\text{int}(f^{-1}(\text{cl}(A)))) \subseteq f^{-1}(\text{cl}(A))$.

(iii) \Rightarrow (i) Let A be an IFCS in Y . By hypothesis $\text{int}(\text{cl}(\text{int}(f^{-1}(A)))) \subseteq f^{-1}(\text{cl}(A)) = f^{-1}(A)$. This implies $f^{-1}(A)$ is an IFSPCS in X and hence it is an IF π GSPCS. Thus f is an IF π GSP continuous mapping.

Theorem 3.30: $(X, \tau) \rightarrow (Y, \sigma)$ be an IF π GPS continuous mapping and $g : (Y, \sigma) \rightarrow (Z, \eta)$ is IF continuous, then $g \circ f : (X, \tau) \rightarrow (Z, \eta)$ is an IF π GPS continuous mapping.

Proof: Let A be an IFCS in (Z, η) . Since g is an IF continuous mapping, $g^{-1}(A)$ is an IFCS in (Y, σ) . Since f is an IF π GPS continuous mapping, $f^{-1}(g^{-1}(A))$ is an IF π GPSCS in (X, τ) . Hence $g \circ f$ is an IF π GPS continuous mapping.

REFERENCES

- [1] Atanassov. K., Intuitionistic fuzzy sets, Fuzzy Sets and Systems, 20 (1986), 87-96.
- [2] Chang, C., Fuzzy topological spaces, J. Math. Anal. Appl., 24, 1968, 182-190.
- [3] Coker, D., An introduction to fuzzy topological space, Fuzzy sets and systems, 88, 1997, 81-89.
- [4] El-Shafhi, M.E., and A. Zhakari., Semi generalized continuous mappings in fuzzy topological spaces, J. Egypt. Math. Soc. 15(1)(2007), 57-67.
- [5] Gurcay, H., Coker, D., and Haydar, A., On fuzzy continuity in intuitionistic fuzzy topological spaces, jour. of fuzzy math., 5(1997), 365-378.
- [6] Hanafy, I.M., Intuitionistic fuzzy continuity, Canad. Math Bull. XX(2009), 1-11.
- [7] Joung Kon Jeon, Young Bae Jun, and Jin Han Park, Intuitionistic fuzzy alpha continuity and intuitionistic fuzzy pre continuity, International Journal of Mathematics and Mathematical Sciences, 19 (2005), 3091-3101.
- [8] S.Maragathavalli and K.Ramesh ,Intuitionistic fuzzy π generalized semi closed sets, Advances in Theoretical and Applied Sciences, 1 (2012), 33-42.
- [9] K.Ramesh and K. Kannan, Intuitionistic fuzzy π generalized semi pre closed sets in intuitionistic fuzzy topological spaces (Submitted)
- [10] Seok Jong Lee and Eun Pyo Lee, The category of intuitionistic fuzzy topological spaces, Bull. Korean Math. Soc. 2000, 63-76.
- [11] Thakur, S.S., and Rekha Chaturvedi, Regular generalized closed sets in intuitionistic fuzzy topological spaces, Universitatea Din Bacau, Studii Si Cercetari Stiintifice, Seria: Matematica, 16 (2006), 257-272.
- [12] Young Bae Jun and Seok- Zun Song, Intuitionistic fuzzy semi-pre open sets and Intuitionistic semi-pre continuous mappings, jour. of Appl. Math and computing, 19(2005), 467-474.
- [13] Zadeh, L. A., Fuzzy sets, Information and control, 8 (1965) 338-353.