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FORMULATION AND STUDY OF ONE BOUNDARY VALUE PROBLEM FOR A THIRD-ORDER EQUATION OF A PARABOLIC-HYPERBOLIC TYPE OF THE FORM $\left(\frac{\partial}{\partial x}-\frac{\partial}{\partial y}+c\right)(Lu)=0$ IN A CONCAVE HEXAGONAL AREA WITH TWO LINES OF TYPE CHANGE

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Annotation. In this paper, one boundary value problem is posed and investigated for a third-order equation of a parabolic-hyperbolic type of the form $\left(\frac{\partial}{\partial x}-\frac{\partial}{\partial y}+c\right)(Lu)=0$ in a concave hexagonal area with two lines of type change.

I. Introduction

Studies of elliptic-parabolic and parabolic-hyperbolic equations of the second order began in the 50-60 years of the last century. In 1959, I.M. Gelfand [1] pointed out the need for joint consideration of equations in one part of the domain of parabolic, and the other part of hyperbolic types. He gives an example related to the movement of gas in a channel surrounded by a porous medium: in the channel, the movement of gas is described by the wave equation, outside of it by the diffusion equation. Then, in the 70-80 years of the twentieth century, research began on equations of the third and high orders of the parabolic-hyperbolic type. Boundary value problems for such equations were posed and studied for the first time by T.D.Juraev [2] and his students [3].

Over the past time, research on boundary value problems for equations of the third and high orders of the parabolic-hyperbolic type has developed broadly, and is currently expanding in the areas of complication of equations and the scope of their consideration, as well as generalization of the problems of equations considered for them (for example, see [4], [5], [6] and etc.).

I. Problem statement

In the plane area xOy the equation is considered

$$\left(\frac{\partial}{\partial x}-\frac{\partial}{\partial y}+c\right)(Lu)=0, \tag{1}$$

where $c\in R$, $Lu\equiv\begin{cases}L_1u\equiv u_{xx}-u_y \ (x,y)\in G_1,\\ L_iu\equiv u_{xx}-u_{yy} \ (x,y)\in G_i \ (i=2,3),\end{cases}$ $G=G_1\cup G_2\cup G_3\cup J_1\cup J_2$, G_1- rectangle with vertices in points $A(0,\,0)$, $B(1,\,0)$, $B_0(1,\,1)$, $A_0(0,1)$; G_2- triangle with vertices in points $A(0,\,0)$, $B(1,\,0)$, $C(0,-1)$; G_3- rectangle with vertices in points $A(0,\,0)$, $A_0(0,1)$, $D_0(-1,\,1)$, $D(-1,0)$; $AB-$ an open segment with vertices at points $A(0,\,0)$ and $B(1,\,0)$; AA_0- an open segment with vertices at points $A(0,\,0)$ and $A_0(0,1)$, т.е. $G-$ a concave hexagonal area with vertices at points $A(0,\,0)$, $C(0,-1)$, $B(1,\,0)$, $B_0(1,\,1)$, $D_0(-1,\,1)$, $D(-1,0)$.

We will divide the area into two parts using a segment AE. Then this area can be written as: $G_2=G_{21}\cup G_{22}\cup AE$, where $G_{21}-$ triangle with vertices in points $A(0,\,0)$, $B(1,\,0)$, $E(1/2,\,-1/2)$; $G_{22}-$ triangle with vertices in points $A(0,\,0)$, $C(0,-1)$, $E(1/2,\,-1/2)$; $AE-$ an open segment with vertices at points $A(0,\,0)$ и $E(1/2,\,-1/2)$.

DOI: 10.48047/INTJECSE/V14I7.276 ISSN: 1308-5581 Vol 14, Issue 07 2022

Now we proceed to the formulation of the boundary value problem for equation (1).

Task $M_{1(-1)c}^{2}$. Find a function $u(x,y)$, which 1) is continuous in a closed domain $\overline{G}$ and has derivatives involved in equation (1) in the domain $G \setminus J_1 \setminus J_2$ and they are also continuous as well as derivatives u_x and u_y are continuous on parts of the boundary of the area G, specified in the boundary conditions; 2) satisfies equation (1) in the domain $G \setminus J_1 \setminus J_2$; 3) satisfies the following boundary conditions:

$$u(1,y)=\varphi_1(y),\quad 0\le y\le 1, \tag{2}$$

$$u(-1,y)=\varphi_2(y),\ 0\le y\le 1, \tag{3}$$

$$u_x(1,y)=\varphi_3(y),\ 0\le y\le 1, \tag{4}$$

$$u\big|_{BC}=\psi_1(x),\ 0\le x\le 1, \tag{5}$$

$$\left.\frac{\partial u}{\partial n}\right|_{BC}=\psi_2(x),\ 0\le x\le 1, \tag{6}$$

$$u(x,0)=f_1(x),\quad -1\le x\le 0, \tag{7}$$

$$u_y(x,0)=f_2(x),\quad -1\le x\le 0, \tag{8}$$

$$u_{yy}(x,0)=f_3(x),\quad -1<x<0 \tag{9}$$

and 4) satisfies the following continuous bonding conditions on the type change lines:

$$u(x,-0)=u(x,+0)=\tau_1(x),\ 0\le x\le 1, \tag{10}$$

$$u_y(x,-0)=u_y(x,+0)=\nu_1(x),\ 0\le x\le 1, \tag{11}$$

$$u_{yy}(x,-0)=u_{yy}(x,+0)=\mu_1(x),\ 0<x<1, \tag{12}$$

$$u(-0,y)=u(+0,y)=\tau_2(y),\ 0\le y\le 1, \tag{13}$$

$$u_x(-0,y)=u_x(+0,y)=\nu_2(y),\ 0\le y\le 1, \tag{14}$$

$$u_{xx}(-0,y)=u_{xx}(+0,y)=\mu_2(y),\ 0<y<1, \tag{15}$$

где $\varphi_i\ \left(i=\overline{1,3}\right)$, $f_k\ \left(k=\overline{1,3}\right)$, ψ_1,ψ_2 – given sufficiently smooth functions, and $\tau_k,\nu_k,\mu_k\ (k=1,2)$ – the functions that are still unknown are quite smooth, and the following matching conditions are met: $\tau_1(0)=\tau_2(0)$, $\tau_1'(0)=\nu_2(0)$, $\tau_1''(0)=\mu_2(0)$.

We prove the following theorem:

The theorem. If $\varphi_1,\varphi_2\in C^3[0,1]$, $\varphi_3\in C^2[0,1]$, $\psi_1\in C^3[0,1]$, $\psi_2\in C^2[0,1]$, $f_1\in C^3[-1,0]$, $f_2\in C^2[-1,0]$, $f_3\in C^1[-1,0]$ and the conditions of approval are fulfilled $f_1(-1)=\varphi_2(0)$, $f_2(-1)=\varphi_2'(0)$, $\psi_1(1)=\varphi_1(0)$, $f_1'(-1)=\varphi_3(0)$, $f_3(-1)=\varphi_2''(0)$, then the task $M_{1(-1)c}^{2}$ admits a single solution.

To prove the theorem by entering the notation $Lu=v$, let 's rewrite the equations (1) in the form of $\frac{\partial v}{\partial x}-\frac{\partial v}{\partial y}+cv=0$. The general solution of the last equation has the form $v=\omega(x+y)e^{-cy}$. Then we get the equation

$$Lu_i=\omega_i(x+y)e^{-cy},$$

where are the designations introduced

$$u(x,y)=u_i(x,y),\ \omega(x+y)=\omega_i(x+y),\ (x,y)\in G_i\ \ (i=1,2,3). \tag{17}$$

The last equation can be rewritten as

$$u_{1xx}-u_{1y}=\omega_1(x+y)e^{-cy},\ \ (x,y)\in G_1, \tag{18}$$

$$u_{ixx}-u_{iyy}=\omega_i(x+y)e^{-cy},\ (x,y)\in G_i\ \ (i=2,3) \tag{19}$$

where $\omega_i(x+y)\ (i=1,2,3)$ – unknown yet quite smooth functions.

If in the equation (19) $(i=2)$ let 's introduce the notation $u_2(x,y)=u_{2k}(x,y)$, $\omega_2(x+y)=\omega_{2k}(x+y)$ $\left((x,y)\in D_{2k}\ \ (k=1,2)\right)$, that equation (19) $(i=2)$ takes the form

$$u_{2kxx} - u_{2kyy} = \omega_{2k}(x+y)e^{-cy} \quad (k=1,2) \tag{20}$$

takes a look.

II. Task research $M^2_{1(-1)c}$

Task first $M^2_{1(-1)c}$ we are exploring in the field of G_2. Given the type of area G_2, we write down the solution of the equation (20) ($k=1$), satisfying the conditions (10), (11):

$$u_{21}(x,y) = \frac{\tau_1(x+y)+\tau_1(x-y)}{2} + \frac{1}{2}\int_{x-y}^{x+y} \nu_1(t)\,dt - \frac{1}{2}\int_0^y e^{-c\eta}d\eta \int_{x-y+\eta}^{x+y-\eta} \omega_{21}(\xi+\eta)\,d\xi. \tag{21}$$

Differentiating (21) по x and y, we will get

$$u_{21x}(x,y) = \frac{\tau_1'(x+y)+\tau_1'(x-y)}{2} + \frac{1}{2}\left[\nu_1(x+y)-\nu_1(x-y)\right] - \frac{1}{2}\int_0^y e^{-c\eta}\left[\omega_{21}(x+y)-\omega_{21}(x-y+2\eta)\right]d\eta, \tag{22}$$

$$u_{21y}(x,y) = \frac{\tau_1'(x+y)-\tau_1'(x-y)}{2} + \frac{1}{2}\left[\nu_1(x+y)+\nu_1(x-y)\right] - \frac{1}{2}\int_0^y e^{-c\eta}\left[\omega_{21}(x+y)+\omega_{21}(x-y+2\eta)\right]d\eta. \tag{23}$$

Condition (6) can be written as:

$$\left.\left(\frac{\partial u_{21}}{\partial x} - \frac{\partial u_{21}}{\partial y}\right)\right|_{y=x-1} = -\sqrt{2}\psi_2(x). \tag{6'}$$

Substituting (22) in and (23) in (2.1.6'), we have

$$-\sqrt{2}\psi_2(x) = \tau_1'(1) - \nu_1(1) + \int_0^{x-1} e^{-c\eta}\omega_{21}(2\eta+1)d\eta,\ 1/2 \le x \le 1.$$

Differentiating the last equality and changing $2x-1$ on $x+y$, we find

$$\omega_{21}(x+y) = -\sqrt{2}\psi_2'\left(\frac{x+y+1}{2}\right)e^{\frac{c}{2}(x+y-1)},\ \ 0 \le x+y \le 1. \tag{24}$$

Similarly as above, we write down the solution of the equation (20) ($k=2$), satisfying the conditions $u_{22}(0,y) = \tau_3(y)$ и $u_{22x}(0,y) = \nu_3(y)$:

$$u_{22}(x,y) = \frac{\tau_3(y+x)+\tau_3(y-x)}{2} + \frac{1}{2}\int_{y-x}^{y+x} \nu_3(t)\,dt + \frac{1}{2}\int_0^x d\eta \int_{y-x+\eta}^{y+x-\eta} e^{-c\xi}\omega_{21}(\eta+\xi)\,d\xi, \tag{25}$$

where $\tau_3(y)$ и $\nu_3(y)$ – unknown yet quite smooth functions.
Differentiating (25) by x and y, we will get:

$$u_{22x}(x,y) = \frac{\tau_3'(y+x)-\tau_3'(y-x)}{2} + \frac{1}{2}\left[\nu_3(y+x)+\nu_3(y-x)\right] + \frac{1}{2}\int_0^x \left[\omega_{22}(x+y)e^{-c(y+x-\eta)} + \omega_{22}(y-x+2\eta)e^{-c(y-x+\eta)}\right]d\eta, \tag{26}$$

$$u_{22y}(x,y) = \frac{\tau_3'(y+x)+\tau_3'(y-x)}{2} + \frac{1}{2}\left[\nu_3(y+x)-\nu_3(y-x)\right] +$$

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$$+\frac{1}{2}\int_0^x\left[\omega_{22}(x+y)e^{-c(y+x-\eta)}-\omega_{22}(y-x+2\eta)e^{-c(y-x+\eta)}\right]d\eta . \qquad (27)$$

Substituting (26) and (27) into (6'), we have

$$-\sqrt{2}\psi_2(x)=-\tau_3'(-1)+\nu_3(-1)+\int_0^x e^{-c(\eta-1)}\omega_{22}(2\eta-1)d\eta,\ 0\le x\le 1/2,$$

a differentiating the last equality and changing $2x-1$ on $x+y$, we find

$$\omega_{22}(x+y)=-\sqrt{2}\psi_2'\left(\frac{x+y+1}{2}\right)e^{\frac{c}{2}(x+y-1)},\ -1\le x+y\le 0. \qquad (28)$$

Now substituting (21) into (5), we come to the relation

$$\psi_1(x)=\frac{1}{2}\left[\tau_1(2x-1)+\tau_1(1)\right]+\frac{1}{2}\int_1^{2x-1}\nu_1(t)dt-\frac{1}{2}\int_0^{x-1}e^{-c\eta}d\eta\int_{1+\eta}^{2x-1-\eta}\omega_{21}(\xi+\eta)d\xi,\ 1/2\le x\le 1,$$

and differentiating this equality, we get

$$\psi_1'(x)=\tau_1'(2x-1)+\nu_1(2x-1)-\omega_{22}(2x-1)\int_0^x e^{-c\eta}d\eta,\ 1/2\le x\le 1.$$

Here changing $2x-1$ on x, we get the first relation between $\tau_1(x)$ и $\nu_1(x)$:

$$\tau_1'(x)+\nu_1(x)=\alpha_1(x),\ 0\le x\le 1, \qquad (29)$$

here

$$\alpha_1(x)=\psi_1'\left(\frac{x+1}{2}\right)+\omega_{21}(x)\int_0^{\frac{x-1}{2}}e^{-c\eta}d\eta .$$

Now we will rewrite equation (1) in the following form:

$$u_{1xxx}-u_{1xxy}+cu_{1xx}-u_{1xy}+u_{1yy}-cu_{1y}=0.$$

In this last equation and in the equation (2.1.20) ($k=1$) В этом последнем уравнении и в уравнении $y\to 0$ and considering the conditions (10), (11), (12), we get the second and third relations between $\tau_1(x)$, $\nu_1(x)$ and $\mu_1(x)$:

$$\tau_1'''(x)-\nu_1''(x)+\tau_1''(x)-\nu_1'(x)+\mu_1(x)-c\nu_1(x)=0,\ 0\le x\le 1,$$

$$\tau_1''(x)-\mu_1(x)=\omega_{21}(x),\ 0\le x\le 1.$$

Excluding from these relations and from (29) the functions $\nu_1(x)$ and $\mu_1(x)$, we come to the equation

$$\tau_1'''(x)+\left(1+\frac{c}{2}\right)\tau_1''(x)+\frac{c}{2}\tau_1'(x)=\frac{1}{2}\alpha_1''(x)+\frac{1}{2}\alpha_1'(x)+\frac{1}{2}\left[c\alpha_1(x)+\omega_{21}(x)\right].$$

Integrating this equation from 0 before x, we obtain the following differential equation

$$\tau_1''(x)+\left(1+\frac{c}{2}\right)\tau_1'(x)+\frac{c}{2}\tau_1(x)=\alpha_2(x)+k_1, \qquad (30)$$

where $\alpha_2(x)=\frac{1}{2}\alpha_1'(x)+\frac{1}{2}\alpha_1(x)+\frac{1}{2}\int_0^x\left[c\alpha_1(t)+\omega_{21}(t)\right]dt$ – known function, a k_1 – unknown yet constant.

When solving equation (30), there can be three cases: 1) $c\ne 2$, $c\ne 0$; 2) $c=2$; 3) $c=0$.

Consider the case 1° ($c\ne 0$, $c\ne 2$). In this case, to find a general solution of equation (30), we must write a general solution of the corresponding homogeneous equation of equation (30). The homogeneous equation of equation (30) has the following form:

$$\tau_{10}''(x)+\left(1+\frac{c}{2}\right)\tau_{10}'(x)+\frac{c}{2}\tau_{10}(x)=0,\ 0\le x\le 1.$$

The characteristic equation of this equation has the form

DOI: 10.48047/INTJECSE/V14I7.276 ISSN: 1308-5581 Vol 14, Issue 07 2022

$$\lambda^2+\left(1+\frac{c}{2}\right)\lambda+\frac{c}{2}=0$$

and its roots: $\lambda_1=-1,\ \lambda_2=-\frac{c}{2}$. Then the general solution of the homogeneous equation –

$$\tau_{10}(x)=C_1e^{-x}+C_2e^{-\frac{c}{2}x}.$$

In this case, we will look for the general solution of equation (30) in the form

$$\tau_1(x)=C_1(x)e^{-x}+C_2(x)e^{-\frac{c}{2}x}, \qquad (31)$$

where $C_1(x)$ и $C_2(x)$ – unknown functions yet.

Differentiating (31):

$$\tau_1'(x)=C_1'(x)e^{-x}+C_2'(x)e^{-\frac{c}{2}x}-C_1(x)e^{-x}-\frac{c}{2}C_2(x)e^{-\frac{c}{2}x}.$$

Selecting functions $C_1(x)$ и $C_2(x)$ so that equality takes place

$$C_1'(x)e^{-x}+C_2'(x)e^{-\frac{c}{2}x}=0, \qquad (32)$$

then we have

$$\tau_1'(x)=-C_1(x)e^{-x}-\frac{c}{2}C_2(x)e^{-\frac{c}{2}x}.$$

Differentiating this equality, we find

$$\tau_1''(x)=-C_1'(x)e^{-x}-\frac{c}{2}C_2'(x)e^{-\frac{c}{2}x}+C_1(x)e^{-x}+\left(\frac{c}{2}\right)^2C_2(x)e^{-\frac{c}{2}x}.$$

And here we select the functions $C_1(x)$ and $C_2(x)$ so that equality takes place

$$-C_1'(x)e^{-x}-\frac{c}{2}C_2'(x)e^{-\frac{c}{2}x}=\alpha_2(x)+k_1.$$

Multiplying (32) by $\frac{c}{2}$ and adding the obtained equality to the last equality, we find

$$C_1'(x)=\frac{2}{c-2}\left[\alpha_2(x)+k_1\right]e^{x}. \qquad (33)$$

Integrating (33) from 0 before x, we will get

$$C_1(x)=\frac{2}{c-2}\int_0^x e^t\alpha_2(t)\,dt+\frac{2k_1}{c-2}\left(e^x-1\right)+k_2.$$

Now substituting (33) into (32), we have

$$C_2'(x)=-\frac{2}{c-2}\left[\alpha_2(x)+k_1\right]e^{\frac{c}{2}x}.$$

Integrating this equality from 0 before x, we find

$$C_2(x)=-\frac{2}{c-2}\int_0^x e^{\frac{c}{2}t}\alpha_2(t)\,dt-\frac{2k_1}{c-2}\cdot\frac{2}{c}\left(e^{\frac{c}{2}x}-1\right)+k_3.$$

Substituting the found function values $C_1(x)$ and $C_2(x)$ in (31), we find

$$\tau_1(x)=\frac{2}{c-2}\int_0^x\left[e^{t-x}-e^{\frac{c}{2}(t-x)}\right]\alpha_2(t)\,dt+\frac{2k_1}{c-2}\left[1-e^{-x}-\frac{2}{c}\left(1-e^{-\frac{c}{2}x}\right)\right]+$$

$$+k_2e^{-x}+k_3e^{-\frac{c}{2}x},\ \ 0\le x\le 1, \qquad (34)$$

DOI: 10.48047/INTJECSE/V14I7.276 ISSN: 1308-5581 Vol 14, Issue 07 2022

and differentiating (34), we get

$$\tau_1'(x)=\frac{2}{2-c}\int_0^x\left[e^{t-x}-\frac{c}{2}e^{\frac{c}{2}(t-x)}\right]\alpha_2(t)\,dt+\frac{2k_1}{c-2}\left[e^{-x}-e^{-\frac{c}{2}x}\right]-$$
$$-k_2e^{-x}-\frac{c}{2}k_3e^{-\frac{c}{2}x},\quad 0\le x\le 1. \tag{35}$$

Substituting (34) and (35) into the conditions

$$\tau_1(0)=f_1(0),\ \tau_1'(0)=f_1'(0),\ \tau_1(1)=\psi_1(1), \tag{36}$$

we find

$$k_3=\frac{2}{2-c}\left[f_1(0)+f_1'(0)\right],\ k_2=f_1(0)-k_3,$$

$$k_1=\frac{c-2}{2\left[1-e^{-1}-\frac{c}{2}\left(1-e^{-\frac{c}{2}}\right)\right]}\cdot\left\{\psi_1(1)-k_2e^{-1}-k_3e^{-\frac{c}{2}}-\frac{2}{c-2}\int_0^1\left[e^{t-1}-e^{\frac{c}{2}(t-1)}\right]\alpha_2(t)\,dt\right\}.$$

Consider the case 2° ($c=2$). In this case, equation (30) has the form

$$\tau_1''(x)+2\tau_1'(x)+\tau_1(x)=\alpha_2(x)+k_1,\ 0\le x\le 1. \tag{37}$$

The characteristic equation of a homogeneous equation corresponding to equation (37) has one twofold root $\lambda_{1,2}=-1$. Then the general solution of the homogeneous equation corresponding to equation (37) has the form

$$\tau_{10}(x)=\left[C_1+C_2x\right]e^{-x}.$$

Then we look for the general solution of equation (37) in the form

$$\tau_1(x)=\left[C_1(x)+xC_2(x)\right]e^{-x}. \tag{38}$$

Differentiating (38), we get

$$\tau_1'(x)=\left[C_1'(x)+xC_2'(x)\right]e^{-x}-\left[C_1(x)+(x-1)C_2(x)\right]e^{-x}.$$

Selecting functions $C_1(x)$ and $C_2(x)$ so that equality takes place

$$C_1'(x)+xC_2'(x)=0. \tag{39}$$

Then we have

$$\tau_1'(x)=-\left[C_1(x)+(x-1)C_2(x)\right]e^{-x}.$$

Differentiating this equality, we get

$$\tau_1''(x)=-\left[C_1'(x)+(x-1)C_2'(x)\right]e^{-x}+\left[C_1(x)+(x-2)C_2(x)\right]e^{-x}.$$

Here we select the functions $C_1(x)$ and $C_2(x)$ so that equality takes place

$$\left[-C_1'(x)-(x-1)C_2'(x)\right]e^{-x}=\alpha_2(x)+k_1.$$

We will rewrite this equation in the form

$$-C_1'(x)-xC_2'(x)+C_2'(x)=\left[\alpha_2(x)+k_1\right]e^{x}.$$

In this case, substituting (39) into the last equation, we obtain

$$C_2'(x)=\left[\alpha_2(x)+k_1\right]e^{x}. \tag{40}$$

Integrating (40) from 0 before x, we find

$$C_2(x)=\int_0^x e^t\alpha_2(t)\,dt+k_1\left(e^x-1\right)+k_3$$

If we substitute (40) into (39), then

$$C_1'(x)=-x\left[\alpha_2(x)+k_1\right]e^{x}.$$

Integrating this from 0 before x, we will get

DOI: 10.48047/INTJECSE/V14I7.276 ISSN: 1308-5581 Vol 14, Issue 07 2022

$$C_1(x) = -\int_0^x te^t \alpha_2(t)\,dt - k_1\left(xe^x - e^x + 1\right) + k_2 .$$

Substituting function values $C_1(x)$ и $C_2(x)$ in (38), we have

$$\tau_1(x) = \int_0^x (x-t)e^{t-x}\alpha_2(t)\,dt + k_1\left[1-(x+1)e^{-x}\right] + \left(k_2 + k_3 x\right)e^{-x},\ 0 \le x \le 1 .$$

Differentiating this equality, we find

$$\tau_1'(x) = \int_0^x e^{t-x}\alpha_2(t)\,dt - \int_0^x (x-t)e^{t-x}\alpha_2(t)\,dt + k_1 xe^{-x} - \left[k_2 + k_3(x-1)\right]e^{-x},\ 0 \le x \le 1.$$

Substituting function values $\tau_1(x)$ and $\tau_1'(x)$ in (37), we find

$$k_2 = f_1(0),\quad k_3 = f_1'(0) + k_2,\quad k_1 = \frac{1}{e-2}\left\{\int_0^1 (1-t)e^t\alpha_2(t)\,dt - e\psi_1(1) + k_2 + k_3\right\}.$$

Finally, consider the case 3° ($c = 0$). In this case, equation (30) has the form

$$\tau_1''(x) + \tau_1'(x) = \alpha_2(x) + k_1,\ 0 \le x \le 1 .$$

Integrating this equation from 0 before x, we get the equation

$$\tau_1'(x) + \tau_1(x) = \alpha_3(x) + k_1 x + k_2,\ 0 \le x \le 1,$$

where $\alpha_3(x) = \int_0^x \alpha_2(t)\,dt$, but k_2 – unknown yet constant.

The general solution of this equation has the form

$$\tau_1(x) = \int_0^x e^{t-x}\alpha_3(t)\,dt + k_1\left(e^{-x} + x - 1\right) + k_2\left(1 - e^{-x}\right) + k_3 e^{-x},\ \ 0 \le x \le 1,$$

where $k_3 -$ unknown constant. Differentiating this, we get

$$\tau_1'(x) = \alpha_3(x) - \int_0^x e^{t-x}\alpha_3(t)\,dt + k_1\left(1 - e^{-x}\right) + k_2 e^{-x} - k_3 e^{-x},\ \ 0 \le x \le 1.$$

Substituting function values $\tau_1(x)$ and $\tau_1'(x)$ in condition (36), we find

$$k_3 = f_1(0),\quad k_2 = f_1(0) + f_1'(0),\quad k_1 = e\psi_1(1) - k_2(e-1) - k_3 - \int_0^1 e^t\alpha_3(t)\,dt .$$

So we found the function $u_{21}(x,y)$ in the area of G_{21} completely.

Now by entering the notation $u_{21}(x,-x) = h_1(x)$ (where $h_1(x)$ – known function), substituting (25) into the condition $u_{22}(x,-x) = u_{21}(x,-x) = h_1(x)$, we come to the equation

$$\frac{\tau_3(0) + \tau_3(-2x)}{2} + \frac{1}{2}\int_{-2x}^0 \nu_3(t)\,dt + \frac{1}{2}\int_0^x d\eta \int_{-2x+\eta}^{-\eta} e^{-c\xi}\omega_{22}(\eta+\xi)\,d\xi = h_1(x),\ 0 \le x \le 1/2 ,$$

where $h_1(x) = \frac{\tau_1(0) + \tau_1(2x)}{2} + \frac{1}{2}\int_{2x}^0 \nu_1(t)\,dt - \frac{1}{2}\int_0^{-x} e^{-c\eta}d\eta \int_{\eta+2x}^{-\eta} \omega_{21}(\xi+\eta)\,d\xi$ – a well-known function.

Differentiating this equation, we obtain the equation

$$\tau_3'(-2x) - \nu_3(-2x) == \int_0^{-x} e^{-c(\eta-2x)}\omega_{22}(2\eta - 2x)\,d\eta - h_1'(x),\ 0 \le x \le 1/2 ,$$

And in this equation I am $-2x$ on y, we come to the first relation between the functions $\tau_3(y)$ and $\nu_3(y)$:

$$\tau_3'(y) - \nu_3(y) = \gamma_1(y),\ \ -1 \le y \le 0 , \tag{41}$$

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where $\gamma_1(y) = \int_0^{-\frac{y}{2}} e^{-c(\eta+y)} \omega_{22}(2\eta + y) d\eta - h_1'\left(-\frac{y}{2}\right).$

Substituting (25) and (5), we get

$$\psi_1(x) = \frac{\tau_3(2x-1) + \tau_3(-1)}{2} + \frac{1}{2}\int_{-1}^{2x-1} \nu_3(t)dt + \frac{1}{2}\int_0^x d\eta \int_{\eta-1}^{2x-1-\eta} e^{-c\xi} \omega_{22}(\eta+\xi)d\xi,\ 0 \le x \le 1/2,$$

and differentiating this, we have

$$\tau_3'(2x-1) + \nu_3(2x-1) = \psi_1'(x) - \omega_{22}(2x-1)\int_0^x e^{-c(2x-1-\eta)} d\eta,\ 0 \le x \le 1/2.$$

Changing here $2x-1$ on y, we get the second relation between the functions $\tau_3(y)$ и $\nu_3(y)$:

$$\tau_3'(y) - \nu_3(y) = \gamma_2(y),\ -1 \le y \le 0, \tag{42}$$

where

$$\gamma_2(y) = \psi_1'\left(\frac{y+1}{2}\right) - \omega_{22}(y)\int_0^{\frac{y+1}{2}} e^{-c(y-\eta)} d\eta.$$

From (27) and (28) we find the following:

$$\tau_3'(y) = \frac{1}{2}\left[\gamma_1(y) + \gamma_2(y)\right],\ \nu_3(y) = \frac{1}{2}\left[\gamma_2(y) - \gamma_1(y)\right].$$

Integrating the first of these equalities from 0 before y, we find

$$\tau_3(y) = \frac{1}{2}\int_0^y \left[\gamma_1(t) + \gamma_2(t)\right]dt + f_1(0).$$

So we found the function $u_2(x,y)$ in the area of G_2 completely.

Now go to the area G_3. Going into the equation (29) ($i = 3$) by $y \to 0$ and changing x on $x+y$, we find

$$\omega_3(x+y) = f_1''(x+y) - f_3(x+y),\ -1 \le x+y \le 0.$$

Now consider the following auxiliary task:

$$\begin{cases} u_{3xx} - u_{3yy} = \Omega_3(x+y)e^{-cy}, \\ u_3(x,0) = F_1(x),\ u_{3y}(x,0) = F_2(x),\ -2 \le x \le 1, \\ u_3(-1,y) = \varphi_2(y),\ u_3(0,y) = \tau_2(y),\ u_{3x}(0,y) = \nu_2(y),\ 0 \le y \le 1. \end{cases} \tag{43}$$

A solution to this problem that satisfies all conditions except the condition $u_{3x}(0,y) = \nu_2(y)$, we are looking for in the form

$$u_3(x,y) = u_{31}(x,y) + u_{32}(x,y) + u_{33}(x,y), \tag{44}$$

where $u_{31}(x,y)-$ solving the problem

$$\begin{cases} u_{31xx} - u_{31yy} = 0, \\ u_{31}(x,0) = F_1(x),\ u_{31y}(x,0) = 0,\ -2 \le x \le 1, \\ u_{31}(-1,y) = \varphi_2(y),\ u_{31}(0,y) = \tau_2(y),\ 0 \le y \le 1, \end{cases} \tag{45}$$

$u_{32}(x,y)-$ solving the problem

$$\begin{cases} u_{32xx} - u_{32yy} = 0, \\ u_{32}(x,0) = 0,\ u_{32y}(x,0) = F_2(x),\ -2 \le x \le 1, \\ u_{32}(-1,y) = 0,\ u_{32}(0,y) = 0,\ 0 \le y \le 1, \end{cases} \tag{46}$$

$u_{33}(x,y)-$ solving the problem

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$$\begin{cases} u_{33xx} - u_{33yy} = \Omega_3(x+y)e^{-cy}, \\ u_{33}(x,0) = 0,\ u_{33y}(x,0) = 0,\ -2 \le x \le 1, \\ u_{33}(-1,y) = 0,\ u_{33}(0,y) = 0,\ 0 \le y \le 1. \end{cases} \tag{47}$$

Here are the functions $F_1(x)$, $F_2(x)$ and $\Omega_3(x+y)$ defined as follows: functions $F_1(x)$ and $F_2(x)$ are known in the interval $-1 \le x \le 0$: $F_1(x) = f_1(x)$, $F_2(x) = f_2(x)$, and in between $-2 \le x \le -1$ and $0 \le x \le 1$ they are still unknown; and the function $\Omega_3(x+y)$ it is defined as follows: in the interval $-1 \le x+y \le 0$ she is famous: $\Omega_3(x+y) = \omega_3(x+y)$, and in between $-2 \le x+y \le -1$ and $0 \le x+y \le 1$ she is still unknown.
The solution of equation (45) satisfying the first two conditions of this problem has the form:

$$u_{31}(x,y) = \frac{1}{2}\left[F_1(x+y) + F_1(x-y)\right]. \tag{48}$$

Substituting (48) into the third of the conditions (45), we find

$$F_1(-1-y) = 2\varphi_2(y) - f_1(y-1),\ 0 \le y \le 1.$$

I'm here $-1-y$ on x, we will get

$$F_1(x) = 2\varphi_2(-1-x) - f_1(-2-x),\ -2 \le x \le -1.$$

Substituting (48) into the fourth of the conditions of the problem (45), we find

$$F_1(y) = 2\tau_2(y) - f_1(-y),\ 0 \le y \le 1.$$

Meaning,

$$F_1(x) = \begin{cases} 2\varphi_2(-1-x) - f_1(-2-x),\ -2 \le x \le -1 \\ f_1(x),\ -1 \le x \le 0, \\ 2\tau_2(x) - f_1(-x),\ 0 \le x \le 1. \end{cases}$$

The solution of problem (46) satisfying the first two conditions of this problem has the form:

$$u_{32}(x,y) = \frac{1}{2}\int_{x-y}^{x+y} F_2(t)dt. \tag{49}$$

Substituting (49) into the third of the conditions of the problem (46), we find

$$F_2(-1-y) = -f_2(y-1),\ 0 \le y \le 1.$$

Here changing $-1-y$ on x, we will get

$$F_2(x) = -f_2(-2-x),\ -2 \le x \le -1.$$

Now substituting (49) into the fourth of the conditions of the problem (46), we find

$$F_2(y) = -f_2(-y),\ 0 \le y \le 1.$$

Means

$$F_2(x) = \begin{cases} -f_2(-2-x),\ -2 \le x \le -1 \\ f_2(x),\ -1 \le x \le 0, \\ -f_2(-x),\ 0 \le x \le 1. \end{cases}$$

The solution of problem (47) satisfying the first two conditions of this problem has the form:

$$u_{33}(x,y) = -\frac{1}{2}\int_0^y e^{-c\eta}d\eta \int_{x-y+\eta}^{x+y-\eta} \Omega_3(\xi+\eta)d\xi. \tag{50}$$

Substituting (50) into the third condition of the problem (47), we get

$$\int_0^y e^{-c\eta}\Omega_3(2\eta-1-y)d\eta = -\omega_3(y-1)\int_0^y e^{-c\eta}d\eta. \tag{51}$$

In the integral standing on the left side of (51), making the substitution $2\eta-1-y$ on $-1-z$, we have

$$\int_{-y}^{y} e^{-\frac{c}{2}(y-z)}\Omega_3(-1-z)dz = -2\omega_3(y-1)\int_0^y e^{-c\eta}d\eta .$$

Differentiating this equality and taking into account this equality again, we find

$$\Omega_3(-1-y) = -3e^{-cy}\omega_3(y-1) - [2\omega_3'(y-1) + c\omega_3(y-1)]\int_0^y e^{-c\eta}d\eta .$$

Now substituting (50) into the fourth of the conditions of the problem (47), we get

$$0 = -\frac{1}{2}\int_0^y e^{-c\eta}d\eta \int_{\eta-y}^{y-\eta}\Omega_3(\xi+\eta)d\xi .$$

Differentiating this equality, we have

$$\int_0^y e^{-c\eta}\Omega_3(2\eta - y)d\eta = -\Omega_3(y)\int_0^y e^{-c\eta}d\eta . \tag{52}$$

Now substituting (48), (49) and (50) into (44), we get

$$u_3(x,y) = \frac{1}{2}[F_1(x+y) + F_1(x-y)] + \frac{1}{2}\int_{x-y}^{x+y} F_2(t)dt - \frac{1}{2}\int_0^y e^{-c\eta}d\eta \int_{x-y+\eta}^{x+y-\eta}\Omega_3(\xi+\eta)d\xi . \tag{53}$$

Differentiating (53) by x, we find

$$u_{3x}(x,y) = \frac{1}{2}[F_1'(x+y) + F_1'(x-y)] + \frac{1}{2}[F_2(x+y) - F_2(x-y)] -$$

$$-\frac{1}{2}\int_0^y e^{-c\eta}[\Omega_3(x+y) - \Omega_3(x-y+2\eta)]d\eta . \tag{54}$$

Assuming in (54) $x = 0$, by virtue of the condition $u_{3x}(0,y) = \nu_2(y)$ and equality (52), after some calculations and transformations, we get

$$\nu_2(y) = \tau_2'(y) + f_1'(-y) - f_2(-y) + \int_0^y e^{-c\eta}\Omega_3(2\eta - y)d\eta .$$

In the integral standing on the right side of this equality, making the substitution $2\eta - y$ on $-z$, we come to the ratio

$$\nu_2(y) = \tau_2'(y) + f_1'(-y) - f_2(-y) + \frac{1}{2}\int_{-y}^{y} e^{-\frac{c}{2}(y-z)}\Omega_3(-z)dz .$$

Differentiating this equality and taking into account this equality again, we find

$$\Omega_3(y) = c[\tau_2'(y) - \nu_2(y) + f_1'(-y) - f_2(-y)]e^{cy} - e^{cy}\omega_3(-y) +$$

$$+ 2e^{cy}[\tau_2''(y) - \nu_2'(y) - f_1''(-y) + f_2'(-y)]. \tag{55}$$

Now in equation (18) assuming $x = 1$, by virtue of (2) and (4), we obtain

$$\omega_{12}(1+y) = [\varphi_3(y) - \varphi_1'(y)]e^{cy}$$

where the designation is entered $\omega_1(x+y) = \begin{cases} \omega_{11}(x+y), \ 0 \le x+y \le 1, \\ \omega_{12}(x+y), \ 1 \le x+y \le 2. \end{cases}$

Now assuming in equations (18) and (19) ($i = 3$) $x = 0$, we will get

$$\mu_2(y) - \tau_2'(y) = \omega_{11}(y)e^{-cy}, \quad \mu_2(y) - \tau_2''(y) = \Omega_3(y)e^{-cy}.$$

Excluding the function from these equalities $\mu_2(y)$, we find

$$\Omega_3(y) = \omega_{11}(y) - [\tau_2''(y) - \tau_2'(y)]e^{cy}.$$

Substituting this in (55), after some calculations, we come to the equation

$$\nu_2'(y) + \frac{c}{2}\nu_2(y) = \frac{3}{2}\tau_2''(y) + \frac{c-1}{2}\tau_2'(y) + s_1(y),$$

where $s_1(y) = c[f_1'(-y) - f_2(-y)] - 2[f_1''(-y) - f_2'(-y)] - \omega_{11}(y)e^{-cy} - \omega_3(-y)$.

Solving this last equation under the condition $\nu_2(0) = f_1'(0)$, we find

$$\nu_2(y) = \frac{3}{2}\tau_2'(y) + \frac{c-2}{4}\int_0^y e^{\frac{c}{2}(t-y)}\tau_2'(t)dt + \beta_1(y),\ 0 \le y \le 1, \quad (56)$$

where

$$\beta_1(y) = f_1'(0)e^{-\frac{c}{2}y} - \frac{3}{2}f_2(0)e^{-\frac{c}{2}y} + \int_0^y e^{\frac{c}{2}(t-y)}s_1(t)dt.$$

Finally, we move to the area. Now we write down the solution of equation (18) satisfying the conditions (2), (10), (13):

$$u_1(x,y) = \int_0^y \tau_2(\eta)G_\xi(x,y;0,\eta)d\eta - \int_0^y \varphi_1(\eta)G_\xi(x,y;1,\eta)d\eta + \int_0^1 \tau_1(\xi)G(x,y;\xi,0)d\xi -$$
$$-\int_0^y e^{-c\eta}d\eta\int_0^{1-\eta}\omega_{11}(\xi+\eta)G(x,y;\xi,\eta)d\xi - \int_0^y e^{-c\eta}d\eta\int_{1-\eta}^1 \omega_{12}(\xi+\eta)G(x,y;\xi,\eta)d\xi. \quad (57)$$

Differentiating (57) by after some calculations, we find:

$$u_{1x}(x,y) = -\int_0^y \tau_2'(\eta)N(x,y;0,\eta)d\eta + \int_0^y \varphi_1'(\eta)N(x,y;1,\eta)d\eta + \int_0^1 \tau_1'(\xi)N(x,y;\xi,0)d\xi +$$
$$+\int_0^y e^{-c\eta}d\eta\int_0^{1-\eta}\omega_{11}(\xi+\eta)N_\xi(x,y;\xi,\eta)d\xi + \int_0^y e^{-c\eta}d\eta\int_{1-\eta}^1 \omega_{12}(\xi+\eta)N_\xi(x,y;\xi,\eta)d\xi. \quad (58)$$

Assuming in (58) $x \to 0$, we get the ratio

$$\nu_2(y) = -\int_0^y \tau_2'(\eta)N(0,y;0,\eta)d\eta + \int_0^y \varphi_1'(\eta)N(0\,y;1,\eta)d\eta + \int_0^1 \tau_1'(\xi)N(0,y;\xi,0)d\xi +$$
$$+\int_0^y e^{-c\eta}d\eta\int_0^{1-\eta}\omega_{11}(\xi+\eta)N_\xi(0,y;\xi,\eta)d\xi + \int_0^y e^{-c\eta}d\eta\int_{1-\eta}^1 \omega_{12}(\xi+\eta)N_\xi(0\,y;\xi,\eta)d\xi. \quad (59)$$

Substituting (56) into (59), after some calculations and transformations we come to the Volterra integral equation of the second kind with respect to $\tau_2'(y)$:

$$\tau_2'(y) + \int_0^y K(y,\eta)\tau_2'(\eta)d\eta = g(y),$$

where $K(y,\eta)$ and $g(y)$ – known functions, and the core $K(y,\eta)$ has a weak feature (with a degree of $1/2$), but $g(y)$ – continuous. Therefore, this equation has a unique solution in the class of continuous functions. Solving this equation, we find the function $\tau_2'(y)$, thus , the functions $\tau_2(y)$, $\nu_2(y)$, $u_3(x,y)$, $u_1(x,y)$.

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