

STUDY ON FLUID DYNAMICAL SYSTEMSONSET OF INSTABILITY IN SOME FLUID DYNAMICAL SYSTEMS

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Abstract

It is well-established in many physical systems across a wide range of length scales that there exist maximum rates at which the wetting and dewetting of solid surfaces are stable. Past this maximum, the flow changes, depositing films on solids (wetting) and entraining the outer fluid into the leading one (dewetting). Many studies have been conducted to better understand the conditions that lead to the emergence of these novel flow states, which may or may not be desirable. Until recently, these changes were captured by steady computations in numerical models. Here, we take a look at the recent developments in computing the time-dependent problem using dynamical systems theory. In this paper, we use a linear stability analysis as a jumping off point to show that unstable solutions serve as 'edge states' that mark the 'point of no return' beyond which fluctuations in the stable flow no longer diminish and, critically, that demonstrate the system can become unstable before the maximum speed is reached.

1. INTRODUCTION

The instability of a liquid-fluid interface as it moves across a solid substrate is referred to as the dynamic-wetting or moving contact-line problem, and it is the focus of this review article. There are numerous fields that rely on fluid motion as their basis, including microfluidics (Stone, Stroock, & Ajdari, 2004), liquid coating & printing (Weinstein & Ruschak, 2004), petroleum recovery (Gerritsen & Durlofsky, 2005), plant protection (Papierowska et al., 2018), groundwater hydrology (Beatty & Smith, 2010), and biological processes (Barthlott & colleagues, 2010).

When the liquid-air-solid contact line moves too quickly, the system becomes unstable, and air is entrained into the coating liquid, leading to low-quality coating films (Miyamoto and Katagiri, 1997; Blake, Dobson, Batts, and Harrison 1995; Blake, Dobson, and Ruschak 2004). Therefore, it is a significant practical challenge that merits scientific investigation to determine when and how these instabilities develop, and hence how to prevent them.

Afkhami, Gambaryan-Roisman, and Pismen (2020), Semenov, Starov, Velarde, and Rubio (2011a), Andreotti, and Snoeijer (2020), and Semenov, Starov, Velarde, and Rubio (2011b) are only a few of the many recent publications that provide reviews on instability. While several computational frameworks, such as Volume-of-Fluid (VoF) (Afkhami, 2022; Fullana, Zaleski, and Popinet, 2020), have been proposed for representing this class of flows, most of the novel approaches to evaluating instability are framework-independent. This fresh interest in the dynamical systems method stems from two seminal works: Severtson and Aidun (1996) used normal-mode analysis to look at two-layer flow in inclined channels, and Christodoulou and Scriven (1988) described how to apply computational dynamical systems theory to viscous flows with free-surfaces. There has been a resurgence of interest in the framework of dynamical systems theory as a tool for understanding transition to instability processes in complicated fluid dynamics systems due to recent breakthroughs in this area. Droplet breakdown (Gallino, Schneider, & Gallaire, 2018); air bubble propagation (Gaillard, Keeler, Thompson, Hazel, & Juel, 2020); and classical turbulence in pipe flow (Eckhardt, Faisst, Schmiegel, & Schneider, 2008) are all examples of such processes. Together, these innovations and improvements in computer efficiency and power have rendered formerly insurmountable challenges manageable. This highlights the importance of refocusing these methods on dynamic (de)wetting events.

Thanks to the contributions of many scientists, including Poincaré (1899) and Kolmogorov (1957), the theory of dynamical systems can look back on a long and storied past. The primary objective is to understand how the system evolves from a certain seed and how certain components, called invariant objects, can influence this transient path (for example, see Kuznetsov, 1998). When discussing fluid dynamics, invariant objects are often referred to as stable states, or non-evolving flow configurations.

Exploring the stability qualities of more exotic invariant objects, including time-periodic flow configurations, is a basic quest made possible by the dynamical systems perspective. The application of this theoretical paradigm to the shifting contact-line problem has enormous potential. Ideas from dynamical systems, such as finding the 'dominant modes' of a system with potentially millions of unknowns, have recently come to the forefront of a number of fields thanks to developments in machine learning and the concept of 'big data,' obtained from numerical simulations and/or experiments (see, for example (among many others), Brunton, Proctor, and Kutz (2016); Brunton and Kutz (2022) and Kutz (2017)). The 'dominant modes' of the dynamical wetting problem can be determined by performing a linear stability analysis with the right set of eigenmodes, which will be discussed in this article.

2. MAXIMUM SPEED OF DE(WETTING)

Here, we'll go through the essential features of the governing equations that cause the RCL/ACL problem to fold bifurcate at a maximum de(wetting) speed. The reader is directed to works such as Vandre (2013), Keeler et al. (2022b), and Sprittles and Shikhmurzaev (2013) for a more thorough mathematical description of the NS.

2.1. Full Hydrodynamic Model

Alternative geometries have also been examined, including curtain-coating (Liu et al., 2019) and liquid bridges (Keeler et al., 2022a), though we explain the notion in the context of a simplified experiment involving flow in two dimensions between two parallel plates. Since asterisks denote all dimensional quantities, it is essential to define certain non-dimensional characteristics up front. The liquid in Figure 1 has a subscript 1, while the gaseous volumes have subscript 2. The Reynolds number, $Re = U^* \rho^* / \mu^*$, the density, ρ^* , and the dimensionless slip length, $\lambda = \lambda^* / H^*$, are all defined relative to the typical speeds U^* and length-scales H^* . Under the assumptions of $Re = 1$ and the Froude number $Fr = U^{*2} / g^* H^*$ (where g^* is the gravitational constant), the Stokes flow equations for both fluid ($i = 1$) and gas ($i = 2$) are satisfied.

$$0 = -\nabla p_i + \delta_i \nabla^2 \mathbf{u}_i, \quad \nabla \cdot \mathbf{u}_i = 0, \quad \delta_1 = 1, \delta_2 = \chi.$$

Where σ is the surface tension coefficient, this forms the dynamic and kinematic boundary conditions for a passive gas at the liquid-fluid interface.

$$\boldsymbol{\tau}_2 \cdot \mathbf{n} - \boldsymbol{\tau}_1 \cdot \mathbf{n} = \frac{1}{Ca} \kappa \mathbf{n}, \quad \frac{\partial \mathbf{r}}{\partial t} \cdot \mathbf{n} = \mathbf{u} \cdot \mathbf{n}$$

where $\boldsymbol{\tau}_i = -p_i \mathbf{I} + \delta_i (\nabla \mathbf{u} + (\nabla \mathbf{u})^T)$ liquid-gas stress tensor, interface curvature, interface outward normal vector (with respect to the fluid phase), and interface position ($\mathbf{r} = (x, y)$) are all dimensionless. We use a Navier slip-model on the walls with a no-penetration condition, i.e.

$$\lambda (\boldsymbol{\tau}_i \cdot \mathbf{n}) \cdot \mathbf{t} = (\mathbf{u}_i - \mathbf{U}) \cdot \mathbf{t}, \quad \mathbf{u}_i \cdot \mathbf{n} = 0,$$

3. RESULTS

The actuator's state-space model, given by equation (1), was used to test our hypothesis. To begin, an AGCN was used to obtain the attention coefficients for each node. The nodes with the highest instability risk were then found across three distinct stability assessments. The first was an examination of spectral stability, the second of topological stability, and the third of symmetry-breaking stability.

3.1. Attention Mechanism

Figure 1 depicts a network with nodes labelled as degree 0, 3, and 7 (degree 4 nodes) and nodes labelled as degree 1, 2, and 6 (degree 2 nodes). Two different strategies were tested. First, the AGCN model was trained to make the distinction between the two types of data. In the second scenario, we introduced noise into the node 0 feature set and used it to train an AGCN model to make the split between the two clusters. To generate the disturbance, we multiplied the feature set by 2. The first two groups of clusters were designated as 0.1 and 0.2.

Training loss versus number of iterations for two cases (without and with perturbation) are depicted in Figures 1 and 2, respectively. Training loss converged to 0.0028 in both cases after 500 iterations, showing that an AGCN model is robust against changes to the input characteristics.

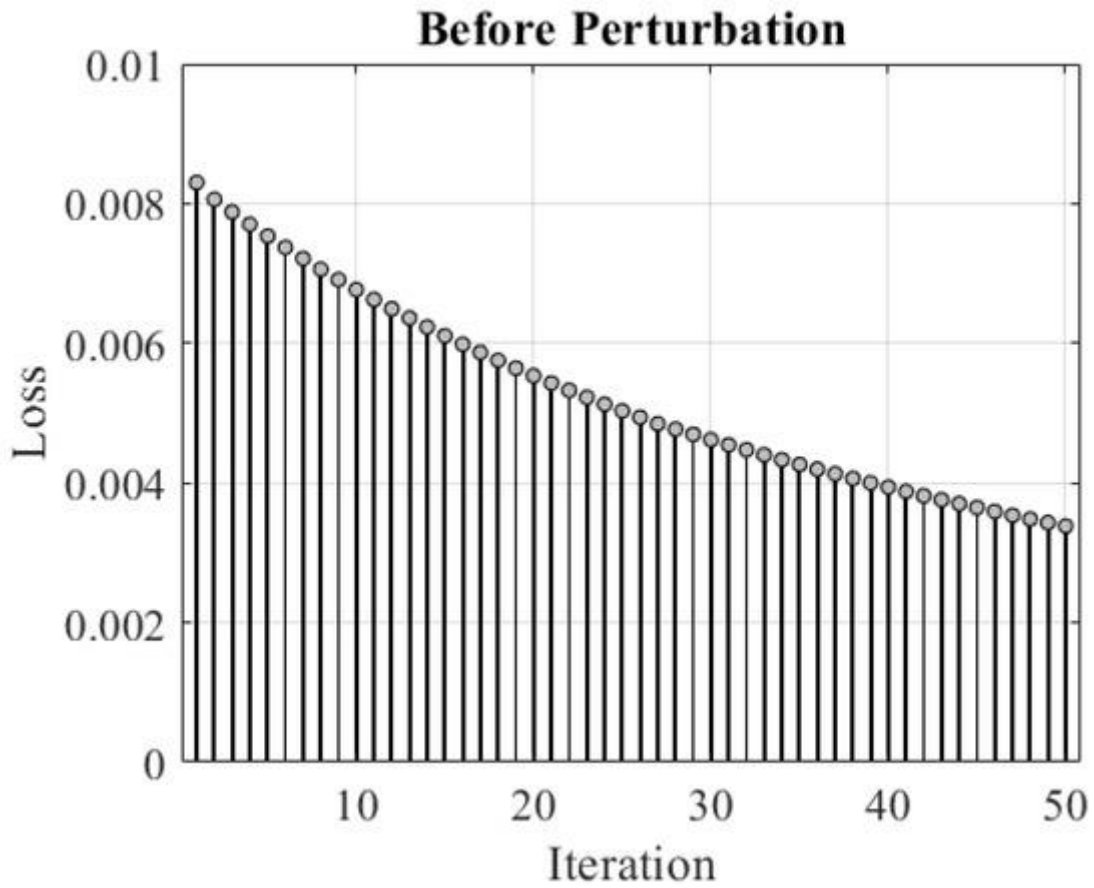


Figure 1. Training error for the no-disturbance case.

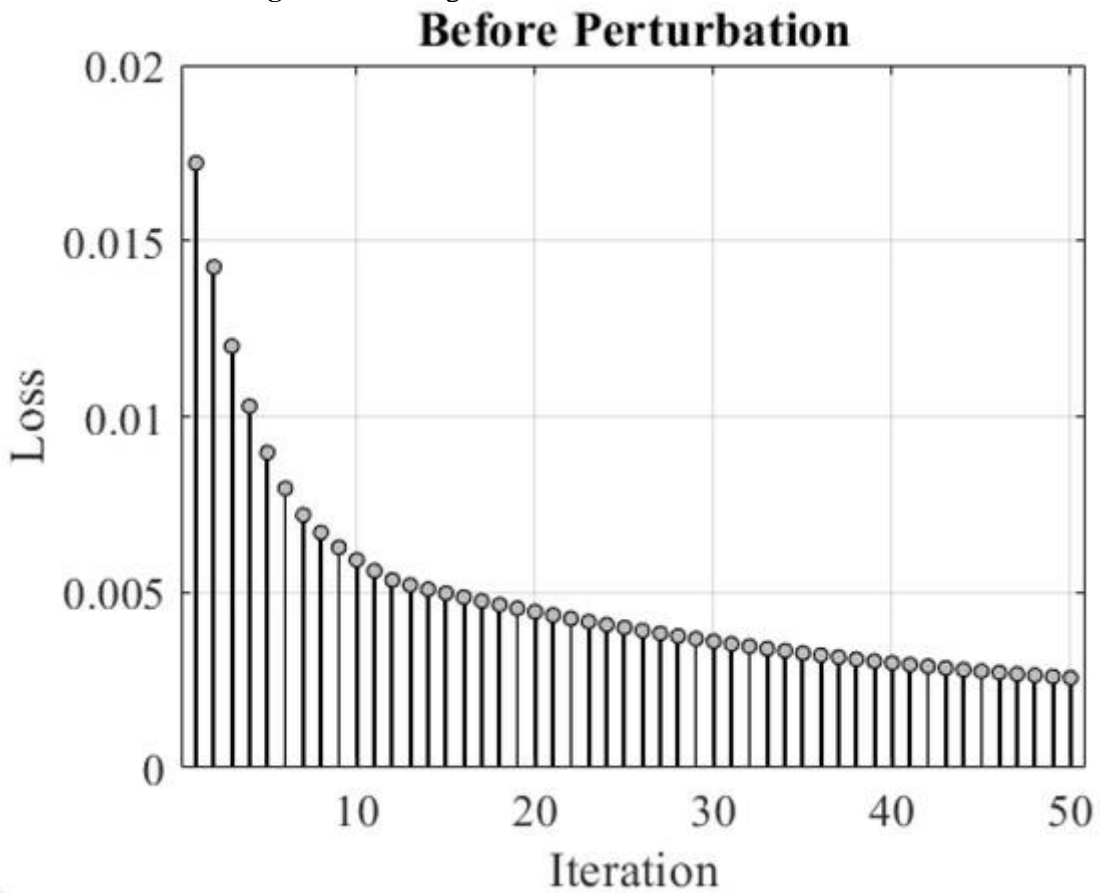


Figure 2. Perturbation-based training losses.

Model predictions are compared with truth labels for the aforementioned cases in Table 2. Table 2 shows that the model predictions made by the AGCN were unaffected by node 1's feature perturbation, demonstrating the model's robustness to feature perturbation.

Table 2. Model prediction of node labels.

Node Id	Actual Label	Predicted Output	
		Without Perturbation	With Perturbation
0	0.01	0.083	0
1	0.20	0.189	1
2	0.20	0.163	2
3	0.01	0.082	3
4	0.01	0.083	4
5	0.20	0.149	5
6	0.20	0.164	6
7	0.01	0.083	7

The nodes with the highest attention coefficients are shown in Figure 3.

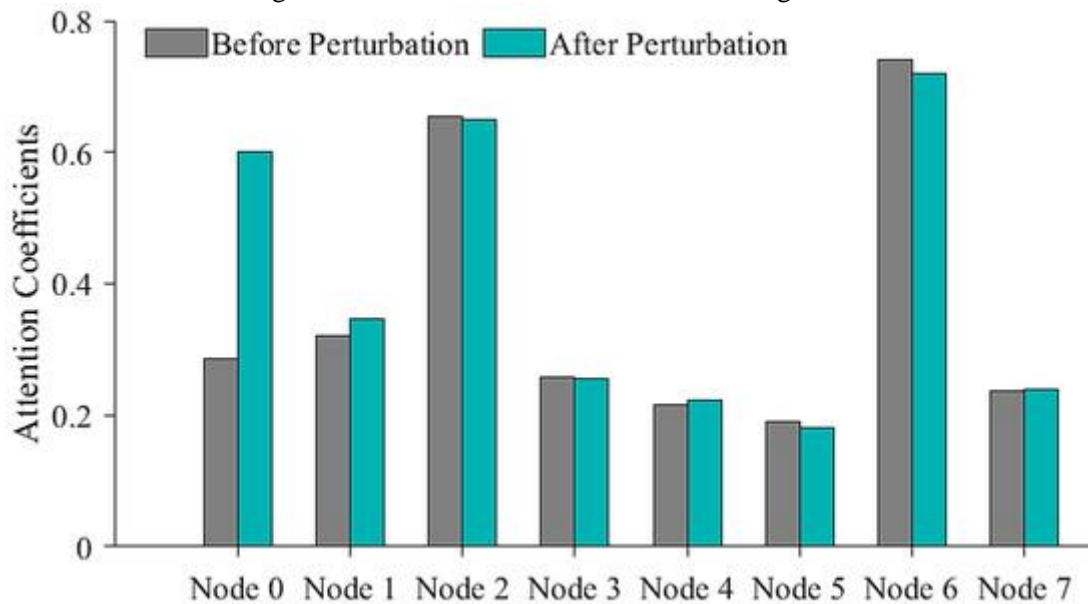


Figure 3. The effect of a perturbation on node 0 is compared to the scenario with no disturbance on node 0. Two and six are the most pressing nodes.

Conclusions

In this research, we showed that there is a triangular relationship between the network's attention mechanism, its instability, and its structural dynamics. We showed that the method by which a machine learning model identifies crucial nodes may be interpreted in terms of the inherent instability of structural dynamics. As a means of making up for the inexplicability of the attention process, we investigated such triangular interactions within a linear dynamical system. We hope to broaden our current research towards nonlinear and nonstationary dynamical systems in future work.

This study's findings shed light on a number of intriguing topics, including: This study, to begin with, demonstrated a connection between attention mechanisms, dynamics, and potentially unstable nodes. It was discovered that the input data to a graph neural network is most useful when it contains

information that can cause a change in the network's dynamics. Through the perspective of instability analysis, this study aimed to explain the attention mechanism. Second, it was discovered that the network's overall dynamic can be altered by the actions of its imbalanced motifs as a whole, suggesting that these motifs merit greater study. Third, we found polarity-driven instabilities in the network's underlying fractal patterns, which redirected our focus from the surface to the underlying structures of polarity transition in our analysis.

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